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THE MATHEMATICS TEACHER

Volume XLIV Number 1

Importance of Mathematics in General Education

By E. R. Breslich
The University of Chicago, Chicago, Illinois

Mathematics, like any other subject, is important to the extent to which it supports and contributes to the purposes of general education. For children are sent to school in order that they become useful members of the community and contribute to the welfare of society. The mathematics referred to in this paper designates that subject matter and training which is important to the entire school population, as distinguished from the special mathematics needed for professional and technical education. It is the mathematics which aids in the preparation of pupils for efficient participation in the activities in which they engage later as adults, and for the assumption of their share of social responsibility.

From the beginning of the American high school to the present time, the teachers of mathematics have insisted that the learner may be benefited greatly by the study of their subject. However, the pupil population of the early high school differed from the present day one in that its members belonged to a rather small and select group. The boys came to school to prepare themselves for leadership in the community. They planned to be ministers, doctors or engineers, which naturally made mathematics an important part of their education. The girls, not to be outdone, preferred the same kind of education. Pupils took school work seriously. Neither pupils nor parents were critical about the courses prepared and recommended by the teachers.

With the growth of the schools, the high school population kept on increasing and toward the end of the nineteenth century approximately half a million pupils were enrolled. Now they formed no longer a select group, and differed widely in interest, industry, and ability. Many "took" mathematics who were not interested and considered it a useless study. Many others were not able to do the work prescribed in the mathematical courses. This part of the school population kept on increasing and created serious problems for the teachers and administrators. The question was raised whether schools should insist that these pupils take the mathematics presented in the courses in algebra and demonstrative geometry. If not, the teachers faced the problem of formulating a more suitable curriculum, one more attractive and more profitable.

The mathematics to be required of all pupils. During the past thirty years, a great deal of work has been done to solve that problem, both by individuals and by

large committees. The aim has been to develop a new course intended to satisfy the needs of those not taking the traditional sequence of algebra and geometry. As far back as in 1923 the National Committee on Mathematical Requirements in recommending such a course says: "The work of the ninth grade should be so planned as to give the best mathematical training which the pupils are capable of receiving in this year and which will be most useful to them with little reference to future courses which they may or may not take." It should be noted that this is not a course for a special group, like "vocational" mathematics, but one containing the mathematics important in the education of all pupils.

The Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics of 1940 made a study of the problem and reached the conclusion that mathematics is important in general education because the average citizen of today needs considerable mathematical knowledge in the activities and experiences of every day life, and because mathematics supplies a means of understanding important aspects of the world. A similar view on the importance of mathematics in general education is expressed by the Commission on Post-War Plans in 1944 and 1945. It recommends a course that develops "mathematical competence" for the ordinary affairs of life as a part of general education, appropriate to the major fraction of the high school population.

As a matter of fact, the citizen of today is dependent on mathematics. He must balance his budget, adjust his expenses to his income, and compute his taxes. As a member of a community or of a state he must participate in community life and should interest himself in the conduct of its business and in its public enterprises. His ability to understand them depends on his mathematical education.

One step in the problem of developing a course containing the mathematics every person should know is to determine the mathematical needs of people in their lives at home and in their every day activities. Studies of this type have taken much time and labor of individuals and committees. They have disclosed considerable agreement. A list of the proposals common to the reports of the National Committee of 1923, the Joint Commission of 1940, the Commission on Post-War Plans of 1944 and 1945, the Committee of Pre-induction Courses of 1943, and the Committee on Essential Mathematics for Minimum Army and Navy Needs of 1943 provides a good outline of the minimum knowledge of mathematics so important to general education that it should be required of all pupils. Such a list contains the following five mathematical requirements:

1. Arithmetical deficiencies of individuals and groups should be removed before the pupil graduates from the high school. This may be accomplished by relating arithmetic closely to algebra and geometry. Such topics as the following should be given special attention: proficiency in the fundamental operations, ratio and proportion, measurement, percentage, tables and verbal problems.

2. An educated person should be familiar with the important properties of the common geometric forms and with indirect measurement.

3. A limited amount of algebra should be taught, including the basic concepts, fundamental operations and simple equations and formulas. Further training in solving problems is essential.

4. There should be instruction in the graphical techniques.

5. Some work in statistics and in numerical trigonometry should be offered.

This is a very conservative program. It comprises the work commonly outlined for junior high school pupils. However, the average citizen of today needs a wider knowledge of mathematics and any school offering such a program should make it possible for those who succeed in the minimum course and who enjoy mathe-

matics to continue further study in the traditional sequence of courses.

The values to be derived from the study of mathematics. In selecting the content of the minimum requirements the committees have taken the "practical" point of view: mathematics is important in general education because people can use it advantageously in modern life; because the newspapers and magazines of today are filled with discussions of a quantitative nature which an intelligent reader is expected to comprehend, but which he fails to appreciate unless he has had the necessary mathematical training; and because people need to interpret complicated numerical data, to understand quantitative studies of social phenomena and to recognize fallacies in conclusions. Hence, high school education should provide pupils with a knowledge of the mathematical concepts, principles and processes which they will need as adults to understand and often to solve problems in which quantitative data and relationships are involved. Many writers and school administrators have rated the utilitarian values as the most important in general education. Indeed, some regard them as the only worth-while values. They would limit mathematical instruction entirely to the practical functions, making it no more than a "tool" subject.

While it is true that the utilitarian values make mathematics indispensable in modern life, the fact should not be overlooked that the study of that subject offers much that contributes to the enjoyment of the learner. Abstract mathematics alone does not hold the interest of many pupils very long, but they will also tire of practical mathematics and applications. This was recognized by the Educational Policy Commission of 1938 when it speaks of the "need of developing an appreciation of the 'cultural values' of mathematics." The National Committee of 1923 goes one step farther and submits a list of the cultural aims including: appreciation of beauty in geometric forms of nature, art and industry; ideals of perfection as to logical structure; precision of statement and thought and logical reasoning; appreciation of the power of mathematics; the role mathematics has played in the development of civilization, in particular in the sciences, industry and philosophy. The parent and the school advisor who tell Mary that there is no sense in her taking high school mathematics for the simple reason that she will probably never use it should give serious thought to her loss of the cultural values. They are depriving her of an enjoyment to which she is entitled, the loss of which she will feel deeply in later years, and for which she will blame them most heartily. Persons with no mathematical training will never reach full appreciation and enjoyment of the beauties of the forms in nature, art and architecture, for they have no knowledge of geometric laws and relationships and cannot recognize even the basic forms. They will always remain curious and mystified by such phenomena as the eclipse, the phases of the moon, and the movements of the planets and stars. They fail to understand our own solar system, to say nothing about the universe. To most people the history of the race is fascinating. Mathematics is closely related to the development of civilization and can never be separated from it. Often civilization stands still waiting for mathematics to advance until it can show the way to proceed. Unlike other innovations, mathematics is not the product of one or a few geniuses. It is the outcome of the labor of many men and women and comes from all parts of the world, but always its contributions have aided the progress of the race.

Teachers have been so busy developing skills and understandings for practical uses that the cultural values have not received the attention they deserve. Fortunately, there is available considerable reading material which pupils may read and enjoy. Such pamphlets as *The Story of Numbers* (American Council of Education) are helpful, and books like the following should be on the shelves of every

high school library: Dantzig's *Number, The Language of Science*, Hogben's *Mathematics for the Million* and Bell's *Men of Mathematics*.

In the early high schools much importance was attached to the "disciplinary" values of all subjects. Thus, it was assumed that the mental training derived from mathematics carried over automatically to other situations, even non-mathematical situations. It was believed that the individual who acquires skill in reasoning in mathematics would be able to reason in any other subject as well; the training in memory obtained in mathematics would be helpful in any situation where memory is used. The development of educational psychology in the early decades of the present century led to many experiments to determine the amount of transfer of training in one subject to other situations. The results were disturbing. It was found that transfer was not nearly as great as had been supposed. The Committee of Ten of 1893 did not yet question the disciplinary values of mathematics and the College Entrance Board a few years later still continued to accept this point of view, but educators began to interpret the small amount of transfer to mean that the idea of transfer may be disregarded as if it did not exist, and that it could be written off as a value of the study of mathematics. Accordingly, the leaders in the reform movements of mathematics which began at the turn of the century kept quiet about the disciplinary values and turned their attention to the utilitarian and cultural aspects of the subject. The reforms advocated by Perry, Klein, Borel and Moore were concerned largely with the selection, organization and presentation of instructional materials.

Later experiments with transfer established the fact that the amount of training depended on certain conditions such as the methods of teaching that were being used, and that under proper conditions mental training in one field could transfer to another. With this came a change in the attitude of educators and psychologists. It is significant that the National Committee of 1923 again classifies the aims of mathematical instruction as utilitarian, cultural and disciplinary, and that it lists as the primary purposes of the teaching of mathematics: powers of understanding and of analyzing quantitative and spatial relationships; habits of thought and of action; insight into and control over environment, all of which are disciplinary aims.

If transfer exists the study of mathematics deserves indeed a very important place in education. Since it is a major function of the school to train pupils in exact thinking the entire school program should contribute to the improvement of the quality of the pupils' thinking. All subjects should provide such opportunities but no subject equals mathematics in developing the power of correct thinking. "Correctness of thinking," says Hutchins, "may be more directly taught through mathematics than in any other way." Mathematics is a way of thinking and can be taught as such. It continually exhibits the processes of thinking in correct simple form, and frowns upon hit or miss methods. It stresses thinking in terms of relationships that exist between facts. The technique of thinking which it uses is the same as that employed in the experiences and relations involved in the social and economic problems encountered by adults in everyday life.

Mathematics lends itself readily to training in reasoning. In daily life, situations constantly arise in which people must proceed with caution and must make careful use of the principles and processes of correct reasoning. They must be able to separate facts from assumptions and to draw correct inferences from them. The reasoning so characteristic of mathematics is essentially the type required in non-mathematical situations where conclusions are to be drawn from given assumptions. Mathematics trains in the processes of drawing correct conclusions. This gives it the right for an im-

portant place in the curriculum of the schools.

However, the teachers should not forget that there is an important "if" attached to the foregoing statements, because transfer does not take place "automatically." Forty years ago Professor Snedden of Columbia University said: "It is useless for teachers to go around talking about the values of mathematics. They exist if and only if they are attained by the pupils." Transfer depends on the way mathematics is taught. "The teacher," says Hedrick, "must teach the pupils how to transfer his training to other and different fields." "A subject must not merely teach particular items," says Judd, "but must stimulate generalization." Thus, there is little training value in having pupils reduce fractions in arithmetic to a simpler form unless they are led to understand the principles underlying this process. Most of the errors and confusion in this particular operation are due to the fact that pupils do not understand the difference between factor and term; have never understood that when a factor of a given number is divided by a second number the given number is divided by the second number; and that when numerator and denominator of a fraction are divided by the same number the value of the fraction is not changed. When the mature mathematician "cancels," he is aware of these principles, but to the pupil not properly taught, cancellation means little more than the crossing out of a number.

In algebra, ability to solve problems does not come automatically from the teaching of specific problems, however many there may be, but from teaching the methods of solving problems through the use of a large variety and number of problems. There is a vast difference between teaching problems and teaching problem solving.

In many schools demonstrative geometry is still taught by the following procedure: first, the pupils study the proofs; then the proofs are memorized; third, the pupils recite the proofs in class; lastly, the pupils are tested by test items calling for the reproduction of proofs. This cannot develop reasoning power. To be sure, memory must always play a part in studying, but emphasis in geometry must be on training in the processes of correct reasoning. Moreover, this must be done in all mathematical situations, and in numerous non-mathematical situations. Since transfer according to Judd is a form of generalization the training must be such as to encourage generalization. That places a grave responsibility on the teacher. He will have to learn how this may be done. Helpful discussions are found in the yearbooks and the journals of mathematics. An excellent piece of work is that of Fawcett in his investigation on "The Nature of Proof" published in the *Thirteenth Yearbook* in 1938. The report of the Progressive Education Committee on the *Functions of Mathematics in General Education* published in 1940 will be most helpful to those who wish to learn more about developing problem-solving ability. The techniques of solving problems, quantitative and mathematical or non-mathematical, encountered by people in every day life are much the same. They call for analyses and careful plans of procedure. Training in attacking and solving problems will be helpful to pupils in and out of school and later in adult life.

For many pupils the power of clear thinking, logical reasoning and problem solving will be of far greater value than a meager knowledge of mathematical facts. The development of these powers makes mathematics more important in general education then the subject itself. "Divested of much of its conventional content and formal organization," says the Commission on the Function of Mathematics in General Education, "mathematics as a mode of thought and an instrument of analysis has an indispensable function in education."

Professor Judd once told a story of a meeting of a commission on teacher train-

ing. The question came up as to the characteristics of a good teacher. One member of the committee suggested that each person present should recall to his mind the best high school teacher he had as a pupil and then determine why he considered him the best. The interesting fact came out that most members named the teacher of geometry, and the reasons were: he taught us how to think; he taught us how to attack problems; he taught us how to reason; etc., all disciplinary values. None mentioned content or even practical applications.

There are other disciplinary values which make mathematics important in education, e.g., effective habits of thought and action; the habit of attacking problems with confidence and assurance; the habit of overcoming obstacles rather than of avoiding them; the habit of carrying tasks to completion; habits of neatness, accuracy, orderliness, attention to detail, observation, reflection, concentration, and constructive imagination, and not the least important, the habit of correct study. All these are important in education. They were always supposed to be by-products of mathematical training and the idea that teachers should and could train pupils is still foreign to many. It is all very well to select better subject matter, to organize it better and to improve presentation, but this is not sufficient. A large per cent of the difficulty with mathematics arises from poor attitudes toward school work. When the teacher reports that John has the ability to do the work but fails because of lack of effort the solution lies in doing something that changes his attitude by arousing genuine interest.

It has been attempted to show that mathematics is worthy of an important place in education. The reason lies in its utilitarian, cultural and disciplinary values. The last two can be attained through improved methods of teaching and do not come automatically from the study of the subject. They must be planned for in the daily work of the teacher. After he has made his plan for the topic to be taught, he must still find a definite answer to the question: where in this plan are the opportunities for deriving the cultural and disciplinary values, and by what method may they be attained by the pupil? Systematic attention to these questions brings surprising results and will hasten the time when the importance of mathematics in general education will not be questioned.

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The Rate of Progress of Pupils in Arithmetic in the Elementary School

By C. C. Grover

Assistant Superintendent of Schools, Oakland, California

The doctrine of individual differences is now generally accepted by educators and its implications are considered in the development of instructional programs in most schools. This doctrine assumes that there is wide variation in abilities, needs, and interests among pupils; that these differences become greater as pupils progress through the grades; and that such differences in ability and interests necessarily result in wide variations in achievement in the various school subjects.

Undoubtedly the most difficult and perplexing problem a teacher faces in a modern elementary school is the adjusting of instruction to pupils differing in the characteristics mentioned above. This problem is present in the teaching of reading, spelling, art, music; in learning to play baseball, in dancing, and, in fact, in every task undertaken in the public schools. The necessary adjustment is particularly difficult in the teaching of arithmetic. It is believed by some psychologists that facility with numerical relationships is a special ability and that satisfactory achievement in mathematics is not only dependent upon general intelligence and previous preparation, but upon this special ability as well. Be that as it may, certainly individual pupils in an arithmetic class differ in intelligence, in the mastery and understanding of number operations, in interest in the subject, in rate of learning, in background of experience, and in many other ways.

Many educators believe that individualizing instruction provides the best solution to this problem. Perhaps, ideally, it does. Such a method of organization and instruction, however, presents many difficulties for most schools. There are many reasons for this. Most of these difficulties depend ultimately upon the cost factor. Individual instruction, to be effective, requires small classes, efficient clerical service, and ample supplies. Since these conditions cannot be met in the usual public school system, other methods of providing for individual differences must be developed. The plan most frequently used is the organization of instructional groups of pupils within the classroom. Under this plan, each instructional group is taught as the abilities and needs of its members seem to require. This is not an unsatisfactory method of teaching. Even with small classes, many teachers prefer it to the individual method of instruction.

Assuming this type of organization, an important question regarding the pupil's rate of progress must be answered—If he is not able to attain the goals for his grade shall he have instruction adjusted to his needs and abilities? On the other hand, if he is above average in his ability to do the work assigned to his grade, shall he be asked to "mark time"? A pupil in the fifth grade, for example, whose ability and previous preparation indicates fourth grade work, is given material at that level; a pupil in the same class, who by the same reasoning should be doing work assigned to the sixth grade, continues to do the work of his class, i.e., the fifth grade. This latter answer, which seems to be the more commonly accepted one, does not seem to be quite fair to the superior student in mathematics.

The reason for this situation is not difficult to find. Teachers and administrators are fearful that pupils leaving a class and teacher where they have been permitted to advance in the textbook or

course of study at varying rates and going to a new class will have such a wide range in achievement as to make instruction for the next teacher very difficult. These difficulties, it is said, will be aggravated when the pupils for the new class come from two or more teachers or when they leave one division of the school system and go into another, e.g., from the elementary school into the junior high school.

There are, of course, elements of truth in this position. It is always easier to assume that all pupils in a given grade or class are equal in ability and achievement than it is to recognize that pupils differ and to provide for these differences. There are some reasonable adjustments, however, which will minimize the difficulties of this wide spread in achievement.

One such adjustment is in the efficient use of the educational materials provided by the school for use in the grades in question. The school can develop an organization so that these materials of instruction will be sent where they are needed and not arbitrarily assigned to a grade or a room. For example, the fourth grade teacher could reasonably expect to have assigned to her room: textbooks used in the third, fourth and fifth grades; flash cards, practice sheets, and drill materials used in these grades; concrete materials, measuring instruments, reference books, and illustrative materials usually assigned to third, fourth, and fifth grade classrooms.

Also the instructional groups within the room can be reduced in number. It is not contemplated that there be eight or ten small groups of pupils in one class working at various places in the curriculum; but rather that there be two or at most three instructional groups within a given room. Under such an arrangement, a group of superior pupils in the fourth grade might have assigned to them the work in the arithmetic curriculum usually given to the fifth grade; if there were only one or two such pupils in the class they would not be given this assignment but their needs would be met in some other manner.

Enrichment? Certainly, but who is to furnish it? To provide enrichment in arithmetic is a much more difficult and time-consuming task than to enrich the language arts curriculum. Materials are available, it is true, but they are much more difficult to locate; they have not been organized for use in a classroom; and the manner in which such materials may be used profitably is not clearly understood. The preparation of elementary school teachers is much less adequate in mathematics than it is in English or social studies. Fewer teachers are intensely interested in mathematics than are interested in the language arts. For these and for other reasons not enumerated, a fair appraisal of the situation leads to the conclusion that in the average elementary school classroom at the present time the teacher is not able to provide an enriched program for superior pupils in arithmetic.

Let it be said immediately that there is hope for the future. Our better teacher-training institutions are aware of the inadequacy of their programs for the training of their students for the teaching of arithmetic and are making plans to improve this condition. As the culture in which we live makes greater demands upon citizens in the understanding of numerical relationships, those responsible for planning education of youth are recognizing the need for a different kind of teaching in the field of mathematics in both the elementary and the secondary schools and much better results have been secured in recent years. It is not unreasonable to expect that the next decade will see even greater improvement in the training and preparation of young teachers than has the last one.

Are we then to cease to attempt to enrich the program in arithmetic? Certainly not. We should put more emphasis upon it. There are many kinds and many levels of enrichment. Superior teaching in one classroom may be enrichment in

another room; enrichment today becomes merely good teaching tomorrow. The position being taken here is not concerned with the amount or desirability of enrichment. That enrichment is desirable is taken for granted. That it constitutes a satisfactory and adequate provision for gifted pupils is another matter and the opinion expressed here is that it does not do so. The problem of enriching the teaching of mathematics in the elementary school deserves more extended treatment than can be given here, but a brief discussion of it will be given. At the outset it might be well to consider some common practices which are not enrichment. The following, in the opinion of the author, definitely may not be classed as in that category:

1. Assigning to superior pupils a larger number of the same kind of examples or problems given to the entire class as a regular lesson.
2. Assigning to superior pupils more examples or problems of the same general type but of somewhat greater difficulty than those given to the entire class in the regular lesson.
3. The practice of assigning superior pupils to assist the less able pupils in regular class work.
4. The preparation of original examples or problems similar to those used in daily assignments and based upon the class text or course of study.

It is believed that true enrichment of mathematics may be found in the following general areas:

1. Greater attention to the understanding of the number system—its origin, development, internal consistency, advantages, etc.
2. Increased emphasis upon the applications of mathematics to problems of society, e.g., relation of assessed valuation to tax rate.
3. Greater facility with mental manipulation of number by means of "short cuts" and other such devices, e.g., use of aliquot parts of the dollar, multiplication of one two-digit number by another two-digit number, checking multiplication and division by "casting out nines," etc.
4. Group projects involving numerical relationships, such as, scale drawing of floor plan of a building in which group is interested and consideration of cost of its construction.
5. Group or individual study and report on such topics as the calendar, origin of various weights and measures, the international date line, etc.
6. Judicious use of games, puzzles, odd and unusual facts, magic squares, and the like, simply for fun. The use of this material in its place and at the proper time will rejuvenate the most moribund class.

In summary, it may be said that differences in ability and achievement are always found among pupils in any class. A skillful teacher will take pupils where he finds them, divide his class into instructional groups as needed, and teach them as seems best in the particular situation. Allowing the better pupils to progress more rapidly through the organized curriculum in arithmetic is a reasonable procedure. There are many sound reasons for such a plan and there seems to be no valid objection to it. Without laboring the point, it seems fair to say that in the average elementary school classroom there is no other adequate method of dealing with the problem of individual differences. With a three to four year range of achievement commonly found in an upper elementary grade classroom, why have instructional groups within the classroom, if the upper group proceeds at the same rate as the middle group? Certainly pupils with superior ability in mathematics profit greatly by the opportunity to progress more rapidly.

What Principles and Applications of Statistics Should Be Taught in High School and Junior College?

By Irving W. Burr

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1. Introduction

A word or two on the nature and importance of statistics might not be amiss. A series of numerical results which vary from measurement to measurement, person to person, place to place, time to time, or object to object are almost certain to be affected and to vary, at least partly, because of random causes or "chance." Such data, whether we like it or not, are statistical data. In analyzing such statistical data and making decisions from them, it is only natural that one should use methods primarily designed for such data, namely, statistical methods. Such methods can minimize the chances of wrong decisions and even estimate the probability of error, and in addition can enable one to learn things from statistical data which would otherwise be missed altogether. Furthermore they can facilitate the design of experiments from which a maximum of information may accrue. Limitations of space preclude the giving of illustrations to show how ubiquitous and important chance variation is. Suffice it to say that statistical methods and thinking are essential to maximum success in fields such as the following: education and psychology, biology, economics, government, actuarial science, manufacturing, and all experimental physical science.

Although statistical background is important to the student, both for his vocation and in his function as a citizen in a democracy, statistics is unfortunately either absent or forms only a minute part of many curricula.

2. Statistics in Our Teaching of Other Subjects

A student once said he liked mathematics because the answer was always perfectly definite. If such is at least partly true in conventional elementary mathematics it cannot be said of statistics. There you get an answer and some measure of its unreliability or "fuzziness." A graduate student, when asked in an oral examination why one might call statistics "fuzzy mathematics," in effect replied, "Because the thinking is unclear." He missed the whole point that so much of science is made up of not what is absolutely true, but only what is probably true with some kind of random error, and that fitted scientific laws are not pure geometrical curves but are bands of probability. Such a concept can help discourage some of the all too common tendency of people to think in absolutes and stereotypes.

The writer believes that it is possible to include some of the statistical viewpoints and even some statistical subject matter in the algebra and trigonometry courses. For one thing it would seem well to use more practical numbers in problems instead of such whole numbers and simplified data that some students immediately begin to look for the error if the answer shows any signs of being complicated. It is natural to have a diameter .321″ and not one quarter of an inch. The writer's plea is that sometimes the problems contain the kind of approximate and rounded off numbers one meets in practical life. Such numbers are statistical in that they involve a random error. There are many opportunities in trigonometry for bringing in the notion of approximate numbers and variation, which opportunities are apparently being increasingly seized upon. Probability chapters have long been in algebra books, and chapters on statistics

are being included sometimes. The main thing in such work is to use data that have meaning to the student such as those to be mentioned later on in this paper. Frequency tabulations; calculations with approximate numbers finding the maximum, minimum and assumed answer; scatter diagrams to study relationships and sampling experiments are examples of items which can be included in an algebra course with reasonable grace.

If statistical thinking and concepts can be introduced into ordinary mathematics courses it is even more true that statistics should be in other courses. For example, we may consider the strength of evidence in history or civics; national statistics and forecasting in economics; variation in laboratory results in chemistry, physics, and biology; probability in heredity; variation in shop performance; variation in test grades, placement scores, etc., and misuses of statistics may be discussed in any course, especially current events or English courses. But for teachers of other subjects to use statistics in the approach to their subject presupposes that they have some statistical background and use it in their work. This poses a gigantic educational problem which will take years to solve. The increasing amount of statistics being taken in colleges and graduate schools will gradually help. Incidentally a knowledge of statistics will help mathematics teachers by giving them a further example of the importance and practicality of mathematics to bring before their students.

3. Introductory Courses in Statistics, High School

It is possible to teach a statistics course to high school students, but it is difficult. If one spends much time on sampling statistics and little time on purely descriptive statistics, the course is likely to be much more lively. But this is difficult because of the early emphasis on descriptive statistics in most present-day texts. The writer does not know of any sufficiently elementary statistics text in which the emphasis is as just suggested.

Another point worth mentioning is that a good understanding of statistical reasoning makes considerable demands upon the maturity of the student. This need not be a crucial stumbling block if it is kept in mind. Every teacher wants to teach the maturer students, but who is going to mature them? It may be a bit like the story of the little red hen.

It is well also to keep in mind that the subject of statistics is most emphatically one of those subjects which can be as dry as dust or fascinating, depending upon how it is presented.

The writer here presents a list of examples and problem material of the kind which will help motivation.

a. Frequency distribution of arrival times of trains or numbers of letters in words for a page.
b. Examination grades, frequency distribution, was one test harder than another, correlation of test grades.
c. Analysis of placement scores, etc.
d. Proportion of children of each sex in families of class members, vs. 50-50.
e. Coin tossing experiments (can easily be overdone).
f. Probabilities in card games expected vs. observed results.
g. Distribution of birth dates through the year and the probability of a coincidence of birthdays.
h. Actual height vs. guessed height for all or some of class.
i. Frequency distribution of coin dates and distribution of sample means from such a ragged population.
j. Prediction of scores for next game of school and distribution vs. actual.
k. Why gambling does not pay, but without so much emphasis that it has the actual effect of popularizing gambling.
l. Ways of estimating attendance at games or population, area sampling.
m. Height vs. weight for class (anonymous data from sheets returned in class).
n. Analysis of laboratory data from a course in the school.
o. Each person make two points on a piece of paper at 3″ as closely as possible without a ruler. Pass in, repeat. Analyze results: correlation, average error, any bias or improvement, any "lone-wolf"?
p. Non-sense correlation, height vs. change in purse. Is there any relationship?
q. Graphs of cost of living, prices, production,

real value of money vs. history. Correlate promising cases.

r. Choose by lot five people in class, note heights, find mean. Repeat building a distribution of means. Compare with the true known mean and standard error of the mean.

s. Accuracy of measurement. Each count number of e's on a page in a reasonable amount of time. Tabulate results and compare with correct result. Curve shape? Repeat on another page. Significant improvement? Continue till learning curve tapers off.

t. Heights of basketball squad vs. school as a whole.

u. Weather data locally. Distribution of cloudiness. At end of a month have each member of class guess at the number of days having precipitation. Any bias? Variability of estimates.

v. Multiplication and division of approximate numbers: the assumed answer, maximum and minimum, number of places to retain, relative error, etc.

w. National polls and sampling.

x. Each member of a class measure three objects with a micrometer and analyze results. Any mistakes? What is the true dimension?

y. Criticism of questionnaires in school, advertisements.

z. Class keep on look-out for misuses of statistics everywhere.

aa. The importance of not thinking in stereotypes. Saying "He is an athlete," says practically nothing about him. The variation may be so enormous that an average or stereotype is meaningless.

bb. Statistical method and scientific method.

cc. Guess answers such as averages, standard deviations, correlation coefficients, significance, etc. in advance of calculation.

4. Basic Courses in Statistics, Junior College

The problem of statistics courses for the junior college is more pressing, because the demand is being more keenly felt there than in high schools. Fortunately the currently available text books are more suitable for such students, because they are more mature, have probably had more mathematics and have taken or are taking concurrently courses such as biology, economics, business and engineering wherein statistical methods can be used. Thus the illustrations of the instructor can be more meaningful.

It is just as important to provide motivation and practical applications as in high school. The subject and its importance must be continually sold to the student. The interest level can reach the heights or the depths, depending on the instructor.

It is my belief that some sort of statistical subject matter now is or will relatively soon be included in the curricula of junior colleges, whether directly in statistical courses or indirectly in other courses. Such an important tool for thought as statistical methods provide cannot be ignored.

5. A Suggested Outline of Topics

The succeeding outline of topics for the statistics course for high school or junior college is to be thought of as one person's idea of what would make a good course:

a. What is statistics? Importance, place in thought

b. Condensing data: frequency tabulation, and graphs. (Cumulative frequency graph later on and only if the need becomes apparent to the class)

c. Mathematical summarization
   i. Averages: mean, median, mode
   ii. Variability: range, standard deviation (mean deviation)

d. Sample vs. population
   i. Introduction by experiments
   ii. Distribution of sample statistics: mean, median, range, standard deviation, proportion
   iii. Tests for compatibility

e. A generalized frequency polygon, the normal curve

f. Testing a hypothesis

g. Significance of differences

h. Correlation: scatter diagram, fitted straight line, correlation coefficient

i. Time series by graphs

j. Review of misuses, statistical thinking and its relation with scientific method

k. Statistics as a field and as a tool

l. Any other topics which one can include in a one or two semester course.

Perhaps the fundamental thing is to bring the students to the point where they will never again look at a number with the same naive, trusting confidence they had as grade school children.

Mathematical Understandings and Judgments Retained by College Freshmen*

By Ben A. Sueltz
State Teachers College, Cortland, New York

Background of This Report

Several years ago the writer undertook a comprehensive study of the measurement of mathematical understandings and judgments at the elementary and junior high school levels. Phases of this study have been described and results have previously been reported.[1] During the past two years the investigation has been carried into the college field. No attempt has been made to evaluate in terms of typical college-level materials. Rather, the materials developed for use in the junior high school grades have been applied to college students in order to determine what types of understandings and judgments have been retained or augmented since their initial period of study. A number of incidental studies such as the relationship of results of tests of understanding and judgment to success in college mathematics and to general college success have been made but the results of these will not be presented at this time. In this report, the implication for shifts in the curricula on the secondary level will be emphasized.

Aims of Instruction and Evaluation

Measurement and evaluation should be held subservient to the valid aims of mathematical instruction. The writer believes that, for the large majority of public school pupils, the major aim of instruction should be one of functional competence in terms of the social, economic, and cultural life of the individual. This in turn implies the abilities: (a) to sense a mathematical situation which may be encountered through various media, (b) to understand the essential mathematics of the situation, (c) to be able to do the necessary thinking and computing to reach a conclusion, (d) and finally the will or feeling of compulsion to act in terms of the mathematics of the situation. This may appear to be a somewhat narrower view of functional competence than that stated by Betz[2] but such is not necessarily the case. It is consistent with the theses of the Commission on Post-War Plans of the National Council.[3]

Measurement in mathematics has tended to be confined to skills and computations and the applications of these skills in written problems. But even the casual observer knows that the mathematics of the real world is very much broader and that it is encountered in many different ways. The exercises used as a basis of this report admittedly have the limitations of removal from the real world because they necessarily have been presented by the "paper and pencil" technique. However, they attempt to place mathematical ideas in functional settings and thereby conjure a real situation in the student's mind.

College Students

The students used in this report are enrolled in a state teachers college. They are average college students; their mean intelligence as measured by the American

* Presented at the Twenty-Seventh Annual Meeting of the National Council of Teachers of Mathematics, Baltimore, April 2, 1949.

[1] See: *The Forty-fifth Yearbook*, Part I, of the National Society for the Study of Education, 1946; *The Elementary School Journal*, February, 1947; and The Mathematics Teacher, October, 1947.

[2] Betz, William, Functional Competence in Mathematics—Its Meaning and Its Attainment. The Mathematics Teacher, May, 1948.

[3] The Second Report of the Commission on Post-War Plans, The Mathematics Teacher, May, 1945.

Council Psychological Examination is slightly above the national average for college freshmen and the standard deviation of their scores is about three-fourths of the national group. Practically all of them have studied at least one year of mathematics beyond grade eight and the large majority have studied algebra and geometry in the ninth and tenth grades. A small minority have had at least three years of mathematics in high school. More than ninety per cent of them are from New York State. The test of Mathematical Understanding was taken by 377 college freshmen and the test of Mathematical Judgment was taken by 323 freshmen and sophomores who were beginning mathematics in college. The percentages shown with the sample exercises are the per cents of correct responses. The exercises selected for discussion were chosen for the purpose of illustrating certain generalizations that are apparent from the tests as a whole.

Mathematical Understanding

Mathematical understanding, as used in this investigation, includes the understanding of concepts, principles, and relationships inherent in a program of meaningful mathematics in the elementary and secondary schools. Not only have the results on tests been considered but many pupils have been interviewed immediately after taking the tests in order to determine if the items were valid for a particular age or grade. The test of understanding contained 53 items. Representative sample items are reproduced on page 15 so that the nature and scope of the test may be judged.

The response to exercise No. 1 should be compared to that on a similar item where students were asked to identify which one of four Roman numeral numbers was incorrectly written and in which only 19% were able to pick the correct answer. We ask whether or not intelligent adults should be able to comprehend these numbers and if so, what would constitute a reasonable per cent? Exercises 2 and 3 deal with fractions. Interviews with students revealed that many of those who chose the wrong answer for No. 2 were timid about the fraction $2\frac{1}{2}/6\frac{1}{2}$. They had never considered that such a fraction might exist and those who accepted it as correct could not see that it equaled 5/13. One might question whether $\frac{4}{5}$ quart (Ex. No. 3), as a bottle, is a real situation. I am told by some students that this is now called $\frac{4}{5}$ gallon. However, this involves a type of mathematical understanding which is not easily comprehended even by high school graduates.

Note the per cents correct for items 4 and 5. They show very low understanding. These exercises are beyond the usual limits of instruction in the elementary school. They are not taught in the typical high school. But they are real in the world of functional uses of percentage. Why doesn't the high school assume responsibility for such learnings?

Item No. 6 is fairly well understood. This is something that most junior high schools teach. Another similar item dealing with the size of arc for a constant angle was not well mastered. Only one student in the more than 90 interviewed had ever seen a cubic foot such as that illustrated in item No. 7! Yet the concept of cubic foot is a very useful functional item. How do schools develop an understanding of cubic foot?

Items Nos. 8, 9, and 10 appear on a part of the test in which each item may have one or more correct answers. These items were scored "number right minus number wrong." That is, if a person chose one wrong answer and one right answer his score was zero. For tabulation purposes and for computing per cent correct, items on which a student received a positive score were counted correct. Students knew there was a penalty for wrong answers.

The concept of "dimension" (No. 8) is very confused. This appears to be true at all levels from grade seven into college. In general "dimension" is confused with "side" in the sense that two lines forming

an angle are believed to illustrate something having two dimensions. Perhaps this is an unimportant item of understanding but it is a common item of the vocabulary of mathematics teachers. Exercise No. 10 seems to illustrate poor thinking and understanding in terms of a comparatively simple algebraic property.

Mathematical Judgment

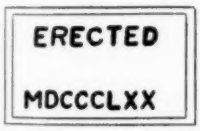
These exercises are designed to require an element of thinking or judgment which goes beyond the simple recognition of some basic mathematical concept or principle. The samples given here were selected to show a variety and range of type and ability.

Note items 1 and 6. Results are about the same. It was noted that many students who correctly estimated the length of the pencil did so by using some measure such as the length of a finger joint. Those who did well on exercise 6 sketched a line to indicate a right angle and then drew their judgment. Two questions arise: First, "Are judgments of this type educationally and functionally significant?" and second, "If some students have learned methods of judgment or approximation, why not teach these to all students?" It is interesting to note that on both of these exercises results for college students are about the same as for pupils in grade 8.

Item No. 2 is interesting for two reasons. College results are very high while correct answers on these same items for students in grades 7, 8, and 9 were less than 50% in each case. In general, junior high school pupils resorted to a great deal of "pencil and paper" work while college students tended to draw their conclusions mentally.

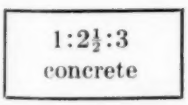
Exercise No. 3 involves a type of mathematical relationship that some elementary school pupils discover independently. Results for grades 7, 8, and 9 showed 11%, 15% and 40% correct respectively. College results are only slightly better. Most wrong answers were "(D) You can't tell." What should the public schools do about the development of such types of mathematical meaning and understanding?

Sample Items from Test on Mathematical Understanding Given to 377 Students and Showing Per Cent of Correct Answers



1. The date on the cornerstone of the old courthouse is given at the left. What year is shown by the Roman Numerals?

(A) 1820 (B) 1870 (C) 1537 (D) 1920 1. *62%*



2. The numbers in the box at the left give the proportions for mixing concrete. This means 1 part cement, $2\frac{1}{2}$ parts sand, and 3 parts of stone. What fraction of the mixture is *sand?*

(A) $\frac{1}{2\frac{1}{2}}$ or $\frac{2}{5}$ (B) $\frac{2\frac{1}{2}}{6\frac{1}{2}}$ or $\frac{5}{13}$ (C) $\frac{1}{3}$

(D) you can't tell exactly 2. *55%*

3. How many bottles like the one at the left can be filled from one gallon of water?

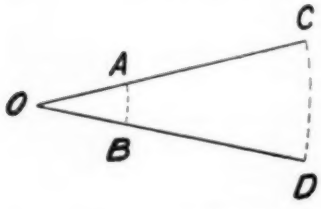
(A) 5 (B) 4 (C) $4\frac{1}{5}$ (D) $1\frac{1}{4}$ 3. *66%*

4. A mixed food for poultry contains "one fourth of one per cent" salt. How is this per cent written as a decimal?

(A) $.0\frac{1}{4}$ (B) .025 (C) .0025 (D) .25 4. *27%*

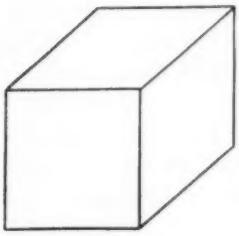
5. A home economics worker noted that large oranges contain 150% *more* juice than small oranges. *How many times as much* juice do the large oranges contain?

(A) 2 times (B) $1\frac{1}{2}$ times (C) $2\frac{1}{2}$ times (D) $1\frac{1}{3}$ times 5. *16%*



6. In the construction at the left, OC is made 3 times as long as OA. The angle BOA has 25 degrees. How large is the angle DOC?

(A) 25° (B) 28° (C) 50° (D) 75° 6. *80%*

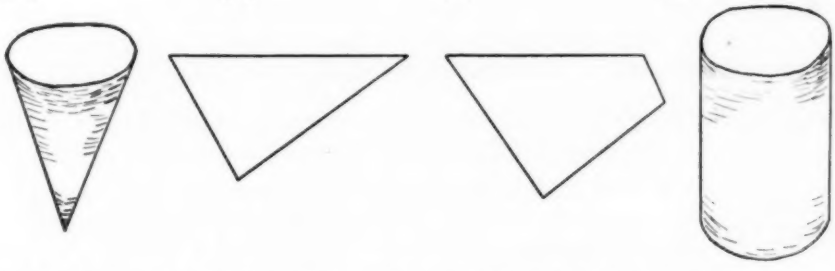


7. The cube at the left represents a cubic foot. If one layer of cubic inches is placed in the bottom of the cubic foot, how many cubic inches would be in the layer?

(A) 12 cu.in. (B) 48 cu.in. (C) 144 cu.in. (D) 1728 cu.in. 7. *54%*

*8. Which of the following objects have two dimensions?

(A) (B) (C) (D) 8. *37%*



*9. In which of the following formulas would the value of the left member be doubled when the value of r in the right member is doubled and other values remain the same?

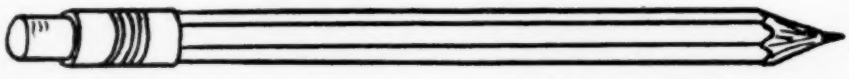
(A) $d=2r$ (B) $c=2\pi r$ (C) $V=\frac{1}{3}\pi r^2 h$ (D) $A=\pi r^2$ 9. *73%*

*10. The root of an equation is: $x=-4$. Which of the following equations based upon this root are correct?

(A) $\frac{1}{4}x=-1$ (B) $4x=16$ (C) $x^2=-16$ (D) $x^3=-64$ 10. *36%*

SAMPLE ITEMS FROM TEST ON MATHEMATICAL JUDGMENT GIVEN TO 323 COLLEGE STUDENTS AND SHOWING PER CENT OF CORRECT ANSWERS

1. How long is the pencil that is drawn below?



(A) $3\frac{1}{2}$ in. (B) $4\frac{1}{2}$ in. (C) $5\frac{1}{2}$ in. (D) $6\frac{1}{2}$ in. 1. *46%*

* From a group, where one or more answers are correct.

2. Elsie plans to make sandwiches like the four shown at the left. How many sandwiches will she get from a loaf that has 20 slices of bread?

(A) 10 (B) 20 (C) 40 (D) 80 2. *72%*

3. Mary was dividing by 28, the first figure in the quotient was 5 and the remainder was 26. What would be the next figure in the quotient after she has "brought down" a figure?

(A) 1 or 2 (B) 4 or 5 (C) 8 or 9 (D) you can't tell 3. *49%*

	seeds	plants
Corn A	5	4
Corn B	10	8
Corn C	50	40

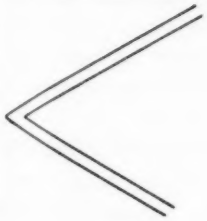
4. The chart at the left shows how many plants sprouted from three different samples of seed corn. Which is most likely to produce 80 plants for each 100 seeds in a bushel of corn?

(A) corn A (B) corn B (C) corn C
(D) all three samples are equally likely 4. *19%*

15 stronger body
30 better tread
15 more traction
OUR NEW TIRE

5. A new tire is advertised using per cents as shown at the left. By what per cent is the new tire better than the old tire?

(A) 60 (B) 20 (C) $22\frac{1}{2}$
(D) you can't tell the per cent exactly 5. *20%*



6. Two roads intersect as shown. Which of the following is the best estimate of their angle of intersection?

(A) 30° (B) 40° (C) 50° (D) 60° 6. *44%*

metal spheres

7. The large sphere has a diameter approximately three times as large as the diameter of the smaller sphere. If the smaller sphere weighs 20 pounds, how much would you expect the large sphere to weigh?

(A) 60 lb. (B) 125 lb. (C) 250 lb.
(D) 500 lb. 7. *17%*

*8. Which of the following have values that can be expressed with whole numbers or decimals so they can be set exactly on an adding or computing machine?

(A) π (B) 7.2 (C) $\sqrt{2}$ (D) $(\frac{1}{2})^2$ 8. *60%*

*9. Mr. Sharpe bought a farm. He said the value of the house was "less than worthless." Which of the following might be the value expressed by Mr. Sharpe?

(A) 1¢ (B) 0 (C) −100 dollars (D) \$0.00½ 9. *37%*

* From a group where one or more answers are correct.

Exercise No. 4 is not valid for junior high school pupils because they failed to sense the element of the relationship between the size of the sample and the reliability of the sample. This was also true for college students. However, if that is an important functional item of learning, it is not reasonable to expect that college students should recognize the situation?

Item No. 5 is interesting and disturbing. Results are only slightly better on the college level than on the junior high school level. This is the type of situation that may be called "sucker bait." Students tend to want to compute when they see numbers and that results in erroneous conclusions. Of course that is what they learned to do in schools.

Several items such as No. 7 have been used with various levels of pupils and the results are uniformly low. These exercises include comparisons of similar objects of both two dimensions and three dimensions. Even college students fail to recognize that this is an application of a proposition which they glibly recite. In the real world these judgments usually must be made in terms of an optical impression; as for example in the buying of fruits. What shall we do about this situation? Demonstrative geometry of the tenth grade seems to have done little or nothing. Of course there are other more meaningful methods of teaching comparisons; why not use them?

Exercises Nos. 8 and 9 may have one or more correct answers. The results for No. 8 are surprisingly good. Many people chose "(B) O" as the answer for No. 9. Some students justified this by the statement that nothing is less than worthless.

Summary

Approximately two thousand junior high school pupils and one thousand college students have taken tests designed to measure the Mathematical Understanding, Mathematical Judgment, and Mathematical Computations in functional situations. These tests have the limitations of the "paper and pencil" technique. Testing in real functional situations would require taking pupils to the grocery store, to street intersections, and to other places where mathematical situations exist. The author has done some of this but has found it rather time consuming. He has also experimented with testing based upon bringing real materials before students. The conclusions here stated are derived not only from the sample items that have been presented but also upon the broader experience in evaluation.

1. Pupils and students at all levels tend to learn those things that the teacher expects of them and to learn in a pattern set by the teacher. For example, a school that glorifies abstract computation of arithmetic and algebra tends to surpass in that type of work and may or may not achieve good results in understanding and judgment. The converse is also true but to a lesser extent because understanding and judgment usually involve some computational elements. The implications for curriculum and for teaching are obvious. What do you want pupils to achieve in mathematics?

2. Weaknesses tend to persist from one level to another. This is particularly true of arithmetic learnings. Pupils who did not comprehend fractions and percentage in the elementary school continue in low comprehension into college. This continuing weakness should probably be blamed upon the school as well as upon the pupil. Mathematics tends to be a segmented group of materials rather than a program of continuous growth in the basic areas of arithmetic, geometry, and algebra.

3. Certain types of exercises in which the mathematical element is not complicated and is found in situations in which older students have had experience prove a good deal easier for the older group. This may be an argument for basing learning upon real experience. It is surprising to find that many college students have had little opportunity to learn from

seeing and handling real things or even the standard models.

4. The extension of learnings in arithmetic is neglected in the typical secondary school program. This is most lamentable if, as most citizens believe, arithmetic is the most functionally useful part of mathematics for most people. Anyone who is acquainted with current arithmetic courses of study knows that these do not attempt to teach all of the arithmetic that the competent citizen requires. Consider percentage: where can a young person enlarge his understanding, extend his computations beyond the simplest cases, and learn to judge in terms of per cent in most of our junior and senior high schools? What is the prime aim of instruction in mathematics if not to provide a basis for more competent and intelligent participation in the civilization in which we live?

5. Schools tend to place too much dependence upon "paper and pencil" work. Many high school and college students set down figures and compute without doing any critical thinking. Furthermore they tend to use pencil and paper for simple things that ought to be done mentally. A considerable part of the mathematics of the real world is encountered orally and visually and requires only a little mathematical understanding upon which a valid judgment or decision should be made. The great dependence upon paper and pencil is not restricted solely to those of lower mental ability.

6. Confusion concerning the purpose of study and the values to be derived from the study of demonstrative geometry persists in the minds of students. In the recognition of geometric figures and concepts in the application of many geometric principles, those students, who studied the informal and experimental type of geometry found in many junior high schools, tend to do as well as many students who spent a whole year at demonstrative geometry.

7. Students tend to do comparatively well in abstract algebra such as computing with negative numbers and in solving comparatively simple equations but they tend also be to little concerned with the understanding and significance of what they are doing. In general "algebraic tricks" rather than algebraic thinking is most characteristic of students who have studied only one year of algebra.

LETTERS

In the February 1949 issue of THE MATHEMATICS TEACHER, Lurline Stewart did a splendid job in regard to the slang expressions used in mathematics classes. She covered all of the pet peeves in the field with one exception. I would like to add that exception to the list.

The slang expression that I find used in all mathematics courses from Intermediate Algebra to Analytic Geometry is "bring over." (Perhaps the reason I do not hear it in any course more advanced than Analytic Geometry is that by the time my students have reached Calculus most of them have been exposed to my teaching and know that "bring over" is taboo.)

The word "transpose" is bad enough, but "bring over" is the most objectionable expression that I have heard used in mathematics. In solving an equation such as $x+4=9$, the students "bring over" the four and find $x=5$. This is all right as long as you do not ask them what they mean, and in equations like the above the slang expression often goes unheeded. But, in equations such as $3x=12$, the students also "bring over" and get the result $x=9$. When asked just what they mean by "bring over," half of the students have no idea and the other half say "transpose."

Now I admit that it is much quicker and easier to say "transpose" than it is to say "subtract 4 from both sides of the equation," but wouldn't it be just as easy and quick and far more meaningful to say "subtraction axiom"?

Is it against the principles of education to teach new words? Why should not students learn new terms in mathematics? Students today are ever increasing their vocabularies with more formidable words than axiom. Most students in high school and college are familiar with such words as atomic bomb and nuclear fission. These terms are much more complex than axiom.

May I add my plea to the plea of Miss Stewart that in high school and college mathematics we omit the slang from our courses and teach the meaningful expressions in their place.

MARGARET F. WILLERDING, Harris Teachers College, St. Louis, Mo.

Mathematics for the Million, or for the Few?*

By William Betz
Rochester, New York

It is now thirteen years since the appearance of a much-debated volume entitled "Mathematics for the Million." That title was decidedly misleading. In reality, the book presupposed a mathematical maturity ordinarily achieved only by "the few." However, it served to focus attention on the real question at issue, namely, what kind of mathematical education should the schools provide not merely for a gifted minority, but also for the average citizen?

Ever since the organization of the International Commission on the Teaching of Mathematics, at the Fourth International Congress of Mathematicians, in Rome, Italy, in 1908, published reports on the mathematical curricula of the participating nations have gradually become available. In the United States, summaries of these documents have appeared in such publications as the National Report of 1923 and the Fourth Yearbook of the National Council of Teachers of Mathematics. They made it quite clear that in the majority of the reporting nations the schools provided one type of education for the masses and a radically different kind of education for the "classes." That sort of program is possible only in a dual system of education consisting of elementary schools for the many and secondary schools for the few.

However, in the United States that arrangement has long been considered undemocratic. Instead, the prevailing policy is that of admitting "all the children of all the people" to every kind of public school. Accordingly, our schools have become genuine educational laboratories on a huge scale. And it may well be that both our successes and our failures in the field of mass education hold valuable lessons for the other nations. In the light of this background, let us briefly examine what has been done here, or is being proposed, by way of answering the question embodied in the title of this paper.

Three principal approaches have been tried or suggested in American secondary schools in their attempts to provide functioning types of mathematical training for *all* American youth.

The *first* is anchored on the popular doctrine that "life situations" should be the primary basis of all curricula. But since mathematics is a *system* of ideas and processes, whereas life situations are incurably *unsystematic*, this approach has failed completely wherever it has been tried. It has always resulted merely in a sort of chaotic "mathematics without mathematics," and it has ignored fundamental aspects of the problem we are considering.

The *second* approach hopes to find dependable answers in the recommendations of authoritative committees and in the techniques of curriculum workshops and laboratories. However, the thousands of mathematical syllabi now crowding the shelves of our curriculum morgues have merely dramatized a hopeless confusion of objectives. And even the reports of *national* committees have regularly been attacked by leading educators as mere reflections of unsupported private opinion.

There remains a *third* approach, often explored partially, but never with anything like scientific completeness or thoroughness. It is that of making a really dependable, full-length study of the role of mathematics in the modern world, from both a practical and a cultural standpoint.

To accomplish that, we should have to look carefully at many aspects of the

* A report submitted to Section VII of the International Congress of Mathematicians, at Cambridge, Massachusetts, September 1, 1950.

question. How and where do we need mathematics in our homes, our vocations and professions, as wage earners, as citizens, as readers, as students, and so on? We should have to find out how mathematics is related to the industrial, commercial, and engineering enterprises of the nation. And in what way do mathematical modes of thinking, attitudes, and types of appreciation permeate the very warp and woof of modern civilization? To obtain the information we need, it would be necessary to analyze the work of many groups of people, such as businessmen, builders, contractors, mechanics, machinists, designers, draftsmen, economists, scientists, statisticians, accountants, navigators, and pilots. When such studies become available, we are sure to discover the utterly untenable character of the idea that only the "few" need genuine mathematical training. On the contrary, it will be found that mathematics is a subject of global significance, and that a citizen of the modern world simply cannot afford to be ignorant of a basic knowledge of mathematics, which must surely involve a little more than grocery store arithmetic and a few rules of mensuration.

It is believed that only on the basis of such objective findings will it be possible to obtain a convincing and enduring foundation for the mathematical curricula of the modern school. It will then be our major task to build a two-track program, or even a three-track program, for the divergent needs of those seeking an immediate life preparation and of those who aim at foundational studies in our colleges and universities.

This brief report is submitted in the ardent hope that it may enlist the active concern of another International Congress of Mathematicians pertaining to this "unfinished business," namely, our common problem of providing more adequate curricula both for the many and the few. Is it too much to expect that a work which was begun so auspiciously in 1908, at the instance of our own David Eugene Smith, be resumed and eventually brought to more complete consummation? It is therefore urged that Section VII of this Congress transmit to the International Mathematical Union an expression of its profound interest in this problem, together with the definite suggestion that proper steps be taken forthwith to initiate its solution.

(At the second session of Section VII, on September 2, Mr. Betz presented the Resolutions which follow. They were adopted unanimously, and will be transmitted to the newly organized International Mathematical Union.)

Whereas the work done by the International Commission on the Teaching of Mathematics, dating back to the Fourth International Congress of Mathematicians in Rome, Italy, in 1908, resulted for the first time in a comprehensive picture of the actual mathematical curricula of the leading nations, and

Whereas a still larger task remained entirely unfinished, namely, that of undertaking a dependable, scientific study of the role of mathematics in the modern world, from a standpoint of both "the many" and "the few," and

Whereas the National Council of Teachers of Mathematics, at its recent meeting (August 21–23, 1950) at the University of Wisconsin, strongly urged that this unfinished business be brought to the attention of Section VII of the present Congress and also of the International Mathematical Union, therefore be it resolved

First, that Section VII of the present Congress gives its unqualified approval to the plan of having such a study made, and

Second, that it wishes to call upon the International Mathematical Union to initiate this program through the continuation and revitalization of the International Commission on the Teaching of Mathematics, helping this Commission to investigate the significance of mathematics in all walks of life, and to make available its findings as a basis for more authoritative and more convincing mathematical curricula throughout the world.

The Use of Simple Algebra in Business Arithmetic

By WILLIAM L. SCHAAF
Brooklyn College, Brooklyn, N. Y.

SOME years ago the writer overheard a conversation between two college students, both majors in mathematics, one of whom was preparing to take an examination for a license to teach business arithmetic. Upon being asked by his companion what he thought of the text he was studying, he remarked that business arithmetic seemed to be a collection of ingenious devices to achieve what might be done much more simply by the use of simple algebraic symbols and equations. While this observation could scarcely serve as an adequate characterization of business arithmetic, it nevertheless suggests that the use of a minimum amount of basic algebra might very well simplify many of the procedures of business arithmetic, and at the same time make them more meaningful.

Before suggesting specific instances where algebraic symbolism could be employed to advantage, let us see how little algebra is actually needed. In the first place, nowadays many elementary school and junior high school pupils, by the end of the eighth grade, have become familiar with simple formulas such as $A=l\times w$, $P=2l+2w$, $C=2\pi R$, $C=\frac{5}{9}(F-32)$, $D=R\times T$, $A=\frac{1}{2}bh$, $A=\frac{1}{2}h(b_1+b_2)$, and possibly a few others. This, of course, is all to the good. Once the learner has begun to understand the meaning of generalized numbers and literal expressions, such as d dollars, t hours, W pounds, h feet and the like, then to learn what simple equations mean and how to solve them is no longer such a formidable task. For most of the purposes to be presented shortly, it is sufficient to be able to solve the following types of equations:

(1) $ax=b$, or $x=\dfrac{b}{a}$.

(If 3 apples cost 12¢, one apple costs $\frac{12}{3}$, or 4¢.)

(2) $\dfrac{x}{a}=b$, or $x=ab$.

(If $\frac{1}{4}$ lb. costs 12¢, one pound costs 4×12, or 48¢.)

(3) $x-a=b$, or $x=b+a$.

(If 3 less than a number is 12, the number is $12+3$, or 15.)

(4) $x+a=b$, or $x=b-a$.

(If 3 more than a number is 12, the number is $12-3$, or 9.)

The transition and extension to these simple equations from the familiar processes of arithmetic is not particularly difficult for most ninth-grade pupils.

Another bit of algebraic technique which is often helpful and not especially difficult is the use of the identity $k(a+b)=ka+kb$, or, if you will, $Km+Kn=K(m+n)$. This may be presented inductively in terms of familiar number relations; e.g., $5\times13=(5)(10+3)=(5\times10)+(5\times3)=50+15$, or 65. In fact, learning this sort of relationship may give added meaning to the multiplication algorism in arithmetic when reteaching the ideas

$$\begin{array}{r} 13 \\ \times\ 5 \\ \hline 65 \end{array} \qquad \begin{array}{r} 10+\ 3 \\ \times\ 5 \\ \hline 50+15=65 \end{array} \qquad \begin{array}{r} 3+10 \\ \times\ 5 \\ \hline 15+50=65 \end{array}$$

of place value and carrying. It is even possible that some pupils may arrive at complete understanding of the process for the first time in their lives.

To illustrate the simplicity, convenience and effectiveness of the use of symbols and equations to indicate mathematical relationships between quantities used in

business, consider first the concepts of *margin* and *profit*. Basically, the important ideas are expressed by

$$S-C=M, \quad \text{and} \quad M=E+P,$$

where $S=$sales, $C=$cost, $M=$margin, $E=$expenses, and $P=$net profit. From these equations it is easy to discover other relationships; for example,

$$S=C+M,$$
$$P=M-E,$$
$$S=C+E+P,$$
$$P=S-C-E.$$

As another illustration, take the troublesome percentage problem, the so-called "Case III," to find the base when the rate and the percentage are known. Pupils should eventually appreciate the fact that the three "cases" of percentage are merely different aspects of but one basic relationship, viz.,

$$P=B\times R;$$

from this, if we are at home with simple algebraic transformations, it is very easy to see that

$$R=\frac{P}{B} \quad \text{and} \quad B=\frac{P}{R},$$

which are "Case II" and "Case III," respectively. If at the same time that we become familiar with such formulas we also get the habit of thinking and writing .06, .35, .8, for 6%, 35%, or 80%, the usual difficulties with Case III will tend to disappear. These relationships frequently arise in connection with the determination of list price, or selling price, or the amount sold on commission, or the additional discount to be offered, etc.

In problems relating to chain discounts, the following formulas should prove useful.

(1) Net cost when two discount rates d_1 and d_2 are offered:

$$C=P\times(1-d_1)(1-d_2),$$

where P is the list price.

(2) Equivalent single discount rate, D:

$$D=1-(1-d_1)(1-d_2),$$

which, like formula (1), can be generalized for any number of successive discounts.

In this connection, a short cut suggested for finding the equivalent single discount rate is to "subtract the product of the two given discount rates from their sum." Thus, discounts of 25% and 20% are equivalent to a single rate of $(25\%+20\%)-(25\%\times 20\%)$, or $45\%-5\%=40\%$. The proof of this rule of thumb method, using formula (2) above, is as follows:

$$\begin{aligned} D&=1-(1-d_1)(1-d_2)\\ &=1-(1-d_1-d_2+d_1d_2)\\ &=1-1+d_1+d_2-d_1d_2\\ &=(d_1+d_2)-d_1d_2. \end{aligned}$$

Admittedly, for a pupil to follow the above derivation, he would have to understand and be able to use negative numbers as well as literal numbers. The need for such skills is frankly not particularly great, and is therefore not urged. However, it is very questionable if a short cut should ever be taught merely as a mechanical or rote procedure without rationalization or proof.

When teaching the topic of simple interest, it is doubtful whether many pupils ever really understand the nature of interest, mathematically considered. By and large it is learned more or less mechanically; thus, $I=P\times R\times T$, and nobody ever questions the significance of multiplying these three factors. To be sure, I is a mathematical function of P, i, and n, to use standard notation. But i is related to I and P, as a parameter: thus, when $n=1$,

$$\frac{I}{P}=i, \quad \text{or} \quad I=Pi,$$

which means that i is simply the fractional part which I is of P. This is an important concept which should be understood; interest at the rate of 4% per

annum means that each year, 1/25 of the principal is due as interest. It is important once more to get into the habit of thinking of i as a decimal fraction, representing so and so many units of money, as well as thinking of it as a per cent. Then relationships and the reasons for the relationships can take on additional meaning. Thus

$$\begin{aligned} S &= P+I \\ &= P+iP \\ &= P+.04P \\ &= P(1+.04) = 1.04P, \text{ or } 104\% \text{ of } P. \end{aligned}$$

When $n=2$, we have:

$$\begin{aligned} S_2 &= P+P(2i) \\ &= P(1+2i), \end{aligned}$$

and, in general,

$$S_n = P(1+ni).$$

In fact, the various relationships involved may be conveniently summarized, symbolically, as follows:

$$I=Pni \qquad S=P+I \qquad P=\frac{S}{1+ni}$$

$$P=\frac{I}{ni} \qquad P=S-I \qquad S=\frac{I}{ni}+I$$

$$n=\frac{I}{Pi} \qquad I=S-P \qquad I=S-\frac{S}{1+ni}$$

$$i=\frac{I}{Pn} \qquad S=P(1+ni)$$

It is to be understood, of course, that there is no thought of having pupils memorize a group of formulas such as this. The desired outcome is an *understanding* of the basic relationships, together with sufficient skill in the use of simple algebraic manipulations, so as to be able to derive and use the appropriate relationship in a given problem situation. Such learning should not and need not be mechanical; approached by teacher and learner in the right way, it can be of invaluable help in all sorts and varieties of problems.

The relationship of interest and interest rate to discount and discount rate is seldom clearly understood by pupils. Even adults frequently labor under the misapprehension that *discount* is simply "interest collected in advance," but a discount rate of 5% is mathematically *not* equivalent to an interest rate of 5% on the same loan for the same period. As a matter of fact, the true discount rate is the reciprocal of the interest rate; true discount is interest on the present value. True discount differs from bank discount in that it is equivalent to interest on a principal, while bank discount is interest on an amount (i.e., principal plus interest).

In order to understand mathematically the concepts of present value and true discount, it is again submitted that algebraic representation should be extremely helpful. Without laboring the point, we shall simply set down the important relationships in such a fashion as to exhibit the analogy between future value, present value, interest and discount. The conventional symbols are:

P = principal, or present value (face value)
S = principal + interest, or future value (maturity value)
I = interest
D = discount
i = interest rate (as a decimal)
d = discount rate (as a decimal)
n = time in years

We then have the following:

(1) $I=Pni$	(1a) $D=Snd$
(2) $S=P+I$	(2a) $S=P+D$
(3) $I=S-P$	(3a) $D=S-P$
(4) $S=P(1+ni)$	(4a) $S=\frac{P}{1-nd}$
(5) $P=\frac{S}{1+ni}$	(5a) $P=S(1-nd)$

Now by equating the right-hand members of equations (5) and (5a), it follows that

$$i=\frac{d}{1-nd}, \quad \text{and} \quad d=\frac{i}{1+ni}.$$

In other words, the interest is the future value of the discount, and the discount is the present value of the interest. It is to be noted also that the expressions $(1-nd)$ and $(1+ni)$ are reciprocals.

Similar observations could also be made with regard to genuine understanding of the nature of compound interest. To be sure, to express these relations mathematically involves the use of exponential notation, which was not included in our original premise of a "few simple algebraic notions." However, without even hinting at the binomial expansion of $(1+i)^n$, or even the mention of logarithms, the *meaning* and *significance* of $(1+i)^n$ as being $(1+i)\cdot(1+i)\cdot(1+i)\cdots$ to n factors can be made clear by simple numerical illustrations, where n is held down to small values. Certainly it should help the pupil to appreciate the contrast between simple and compound interest if he can see the mathematical difference between the expressions

$$P(1+ni) \quad \text{and} \quad P(1+i)^n.$$

How far this approach can be carried must be left to the judgment of the individual teacher.

We suggest without further comment a few other relationships in which algebraic expression could help understanding.

(1) The *cost of goods sold* during a given period:

$$C=I_1+P-I_2,$$

where I_1 = initial inventory, I_2 = final inventory, and P = purchases during the period.

(2) *Average inventory*:

$$I=\frac{I_1+I_2}{2}.$$

(3) *Annual depreciation* (straight line method):

$$D=\frac{C-S}{n},$$

where C = initial cost, S = scrap value, and n = estimated life of asset in years.

(4) *Book value* (straight line depreciation) after k years:

$$B_k=C-\frac{k(C-S)}{n}.$$

(5) *Amount paid on a fire insurance claim* under an 80% coinsurance clause:

$$A=\frac{PL}{.8V},$$

where A = amount paid on claim, P = face value of policy, V = value of the property, and L = amount of loss as determined by the insurance adjuster.

Discussion of the above formula (5) reveals that it holds only for values of P equal to or less than $.8V$, since in no case will the company pay more than the face value of the policy.

In conclusion, we urge once more that a greater disposition to use algebraic formulation to express quantitative relationships has much to commend it, provided, however, that such learning is meaningful rather than mechanical or superficial.

Plan to Attend
The Twenty-Ninth Annual Convention of the National Council, Hotel William Penn, Pittsburgh, Pennsylvania, March 28-31, 1951

A very good program has been arranged, including visiting Pittsburgh schools on Wednesday and Thursday, sightseeing trip on Thursday, a big general meeting on Thursday night with a reception following, banquet on Friday night and the convention luncheon on Saturday at noon. A full schedule of meetings, discussions, exhibits, films, demonstrations, and panels has been arranged for Thursday, Friday, and Saturday which should appeal to elementary, secondary, and college teachers and to supervisors and administrators. The complete program with announcements and preregistration form will appear in the February issue of the *Mathematics Teacher*.

Know Your National Council Representative

By KENNETH E. BROWN

Chairman of State Representatives, Department of Mathematics, University of Tennessee, Knoxville, Tennessee

MISS ANNIE JOHN WILLIAMS

NORTH CAROLINA REPRESENTATIVE

MISS ANNIE JOHN WILLIAMS, a high school teacher at Julian S. Carr Junior High School, Durham, North Carolina, has been an active member of the National Council of Teachers of Mathematics for twelve years. She reports that she finds the articles in THE MATHEMATICS TEACHER very helpful to her in teaching students in the junior high school. Her interest in the improvement of the teaching of mathematics is indicated by her attendance for six years at the Mathematics Institute at Duke University. The sole purpose of this Institute is to provide a mathematics laboratory that will contribute to the improvement of mathematics education. Her special interest in the improvement of the teaching of mathematics is in harmony with her work as National Council Representative. Each year she writes several hundred letters to teachers of mathematics asking them to join the Council—the only national organization devoted solely to their cause. Miss Williams will welcome the assistance of the mathematics teachers in North Carolina in helping her with the National Council work.

ARIZONA REPRESENTATIVE

Mr. Louis Leithold, Instructor in Mathematics, Phoenix College, is a native of California. He received both his B.A. and M.A. from the University of California at Berkeley and joined the faculty of Phoenix College in 1946. The following year he became the National Council Representative in Arizona. Although the number of teachers of mathematics in Arizona is not large, this group has organized the Arizona Mathematics Association which meets twice each year. This increased interest in mathematics education is also evidenced by the steady increase in the National Council members in Arizona.

MR. LOUIS LEITHOLD

The Affiliated Groups and the Delegate Assembly

JOHN MAYOR, *Chairman*

The Committee on Affiliated Groups

THE Affiliated Groups of The National Council of Teachers of Mathematics will be interested in making preparations during January, February, and March for the coming National Council meeting in Pittsburgh. Two meetings of the Delegate Assembly have been scheduled. The first meeting is on March 29, 8:00–10:00 A.M. and the second on March 30, 1:15–2:15 P.M. The Delegate Assembly will be concerned with relationships between the Affiliated Groups and the National Council. There will also be a section on Affiliated Groups at the Pittsburgh meeting. In this section a panel of presidents of Affiliated Groups in the Pittsburgh area will lead a discussion on the programs and plans of the various Groups in their own localities.

The official delegate of each Group will want to become familiar with the issues which will be discussed at the Delegate Assembly and the point of view of the Group, which he represents, on these issues. Through the *Newsletter of Affiliated Groups*, the Groups have been invited to submit problems for discussion early enough to make sure that it would be possible for each Affiliated Group to have considered these problems before the Assembly meeting. The names of the official delegates should be in the hands of the chairman of Affiliated Groups by February 1, 1951.

Questions which have been suggested for discussion at Pittsburgh, and on which the Delegate Assembly may wish to make recommendations to the Board of Directors or to the business meeting, are as follows:

1. Should any of the decisions of the First Delegate Assembly, as reviewed in the November issue of THE MATHEMATICS TEACHER, be re-examined in the light of the past year's experiences, to determine if additional action is desirable at this time?
2. By what means can the National Council better serve the various regions and Affiliated Groups?
3. How can a closer relationship be established between the program of State Representatives and the program of Affiliated Groups?
4. What means of determining problems for discussion and what procedures in the Delegate Assembly will serve the interests of the National Council most effectively?
5. Would a plan of rotation for publication of the *Newsletter of Affiliated Groups*, such as is practiced in the publication of the periodicals of some of the Groups, be feasible and how can this be worked out, if found feasible?
6. Does your Group need the services of a Speakers Bureau, and if so, how can such a Bureau best serve your Group and the cause of the improvement of mathematics instruction at all levels?
7. How can Affiliated Groups best contribute to the total program of the National Council and especially to the programs of the meetings, to its publications, to its investigational activities, to the promotion of good public relations, and to growth in membership?
8. Should the name of the National Council be changed? The vote at the First Delegate Assembly favored a change in name, but further consideration, especially of possible new names, seems desirable. Some of the suggested names are Council of Teachers of Mathematics, International Council of Teachers of Mathematics, and American Council for Mathematics Education.
9. Would the Affiliated Groups be interested in a cooperative plan for contest sponsorship?
10. How can Affiliated Groups arrange for sharing and exchange of studies in progress, teaching materials prepared, and any other activities which have been found especially valuable?
11. How can the National Council more effectively carry out its purpose to improve mathematics teaching at all levels of instruction?

The President's Page

You will be interested in knowing that your National Council has taken the first step in a big venture, an endeavor that will ultimately call for the help of individual members and of the affiliated groups.

For a long time our leaders have been telling us that we are many years behind in our mathematics education in this country, and that we are due for a reform. We recognize that all is not well with our programs of mathematics and mathematics teaching, and we are anxious for improvements to be made. Many have been saying that the National Council should be doing something about it and are looking to the National Council for leadership. A complete over-all study of the situation is suggested, the purpose being to bring about this much needed and long overdue reform. The problem is a huge one, an endeavor that will require the cooperation of several national organizations, committees, and agencies. The necessary study and research will require great financial support and will call into service the most capable people that can be found. It will take careful planning, and the work will extend over several years.

Despite the vastness of the task, it should be done. Some beginning should be made immediately. Some group of national repute and influence should take the initiative in the matter and suggest a plan of action. With this in mind and completely desirous of having the National Council do its part, the Board of Directors, meeting last August in Madison, authorized the appointment of a committee of five, this committee to be known as the Research Committee. The committee was appointed early in October and charged with the responsibility of studying the problem and making recommendations to the Board. The committee is a permanent one, membership to be increased or changed as deemed advisable. The committee made a preliminary report to the Board at the Florida meeting in December. What can and will be done remains to be seen, but we are depending upon the Research Committee to point out the possibilities and to develop plans whereby the National Council can participate in a long-range program of study and research on the problem. From the information thus gained, we would strive for the needed changes and improvements.

Indications are that the work of this Research Committee will bring about cooperation with other groups, such as the Policy Committee which is composed of representatives of the American Mathematical Society, the Mathematical Association of America, the Institute of Mathematical Statistics, and the Association of Symbolic Logic. This Committee is interested in the welfare of mathematics and mathematics teaching and we hope to have a representative of the National Council on this committee soon. It may very probably mean that the Cooperative Committee on Science and Mathematics Teaching of the American Association for the Advancement of Science, a committee on which the National Council has been represented for several years, will lend its support. It may lead to our being represented on the National Science Foundation, an organization whose members are appointed by the President of the United States, and which is now in the organizational stage. The Office of Education of the Federal Security Agency is showing a definite interest in mathematics and may soon have on its staff a specialist in mathematics. From such relationships and those of the National Education Association, it is highly probable that contacts with foundational grants can be made to support a research project such as the Research Committee will advise. The Mathematical Association of America and the National Council of Teachers of Mathematics have cooperated before on major endeavors and have produced valuable

results. It is time for them to do it again. The outlook for cooperation with other interested groups is good and the prospects for securing financial backing is promising.

Eventually many will be called upon to help with this project. The Research Committee may be enlarged and sub-committees formed. Let the members of the committee know that you are interested and willing to help. Write to them and give them the benefit of your ideas and suggestions. It is a huge task and they need your help. The members of the committee are: Chairman, Henry Van Engen, State Teachers College, Cedar Falls, Iowa; Howard F. Fehr, Teachers College, Columbia University, New York 27, New York; John J. Kinsella, New York University, Washington Square, New York 3, New York; F. Lynwood Wren, George Peabody College for Teachers, Nashville, Tennessee; Lenore John, The Laboratory School, University of Chicago, Chicago, Illinois.

H. W. CHARLESWORTH, *President*

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
State Teachers College, Oneonta, New York

THE ART OF RECKONING. III. WHAT YOU CAN DO IF YOU KNOW THE DOUBLES

THE USE of doubling as a method of multiplication appears in the Ahmes Papyrus (*c.* 1650 B.C.) in Egypt. Together with finding half of a number (mediation), doubling (duplication) was ranked with addition, subtraction, multiplication, and division as a fundamental process in the work of the Englishman Sacrobosco in the thirteenth century. It has been associated with mathematical puzzles and through its connection with the binary series 1, 2, 4, 8, 16, . . . , it is related to the electronic computing machines.

When multiplying by the doubling process, the computer tacitly recognizes that any whole number whatever may be broken into parts that are numbers taken from the binary series. Thus the number 49 is made up of 32, 16 and 1. To multiply 103 by 49, the computer would write

1—	103
2	206
4	412
8	824
16—	1648
32—	3296

Notice that the multiplicand 103 is in line with the number 1; the double of 103 is in line with 2; the double of the double (i.e., four times) is in line with 4 and so on. Then since 49 is 32 and 16 and 1, the product of 49 and 103 is 32×103 plus 16×103 plus 1×103. So the computer checks the numbers opposite 1, 16 and 32 and then adds.

1	103
16	1648
32	3296
	49×103 is 5047

In the Ahmes Papyrus, the details of the multiplication process are given in some cases, omitted in others. Multiplication as it appears in this document is summarized as follows:

"Direct multiplication by integers was generally confined to the multipliers 2 and 10; that is, in cases where the process was explicit, and it may be presumed that in all cases, they multiplied by repeated doubling, or occasionally multiplying by 10, and adding the products formed from multipliers that would make up the given multiplier."*

* *The Rhind Mathematical Papyrus*, vol. I, Arnold Buffum Chace, Mathematical Association of America, 1927, p. 2.

The products of 19 by 6 and 23 by 12 are given as follows:

	1	19	1	23
	2	38	10−	230
	4−	76	2−	46
Total	6	114	12	276

The Ahmes Papyrus seems to consider division as a second type of multiplication in which the product and multiplicand are given and in which the multiplier is to be found. For clarity, we shall use the terms of division. The writer of the papyrus suggests that the computer is to use successive doubles of the divisor and its multiple of 10 and from these to build up the dividend as nearly as possible, and then find the remainder needed to complete the dividend. It is easier to do this than to describe it. For example, to divide 657 by 34, we would proceed as follows:

Table of doubles		subtracting:	
1	34		657
2	68	16×34	544
4	136		113
8	272	2×34	68
16	544		45
		1×34	34
			11

Thus $657=(16+2+1)34+11$. Accordingly the quotient is $(16+2+1)$ or 19 and the remainder is 11.

The fact that any whole number may be expressed as the sum of terms chosen from the series 1, 2, 4, 8, . . . has a number of applications. One is the number puzzle that asks how one hundred ducats could be placed in seven bags in such a way that any number of ducats up to 100 could be paid out without opening any of the bags. The solution was to place one ducat in the first bag, two in the second bag, four in the third, eight in the fourth, sixteen in the fifth, thirty-two in the sixth and all the rest in the seventh.

A more practical application of the use of the series is in the manufacture of the weights formerly used by goldsmiths and others. The weights were made in the value of 1, 2, 4, 8, . . . units and the fractional weights were $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc. The appearance of these weights was interesting. The smallest one was a flat cylinder of metal which fitted closely into a cylindrical cup which was the next larger weight. This in turn fitted into another cylindrical cup until the largest of the series was the case for the entire set and often took the form of a sort of metal pail with a hinged cover.

The goldsmiths' weights were the basis of a number puzzle which asked the very least number of weights that could be used to weigh any whole number of ounces from 1 to 40. By the device of putting the weights on either side of the scales at will, the weights 1, 3, 9, 27 would be sufficient. To weigh two ounces, the one ounce weight would be on the side of the object to be weighed and the three ounce weight on the other side. To weigh five ounces, the nine ounce weight would be on one side, the one ounce and the three would be on the side of the thing to be weighed.

A modern puzzle use of the binary series is in a series of cards with the number 1, 2, 4, 8, 16, . . . one at the top of each card. Then all the odd numbers up to the desired limit are written on the card labelled 1. On the card marked 2 are listed all the numbers which require 2 in their composition. Thus for the numbers from 1 to 16 the cards would appear in this form:

1	2	4	8
3	3	5	9
5	6	6	10
7	7	7	11
9	10	12	12
11	11	13	13
13	14	14	14
15	15	15	15

Then to find the number of which a person is thinking, ask him to select the cards that have that number printed on them. Add the numbers at the top of the cards that are selected and this will certainly be the number the other person had in mind.

A later article in this series will discuss the uses of the binary series in computation.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
*Monroe High School, St. Paul, Minnesota**

THIS section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

A NOTE ON THE USE OF FELT FOR THE MANUFACTURE OF VISUAL AIDS

Two reasons for the infrequent use of visual aids in the teaching of mathematics are (a) the length of time required for their manufacture compared with the brief space of time for which they are used in the classroom, and (b) the lack of adequate storage space for such apparatus, which must obviously be large enough to be seen by the "back row" boys. Often the choice of a suitable material eliminates these difficulties.

It is a matter of common experience that a piece of wool will readily stick to a rough tweed jacket or any piece of woolen material. In the same way, quite large pieces of felt placed face to face will adhere. If a drawing board covered with black felt is used as a background and the board held vertically for display to the class, then other pieces of brightly colored felt can be placed thereon. Once placed, these will neither fall nor slip, and with their use the whole range of area properties can be demonstrated—triangle, parallelogram, trapezium, circle, Pythagoras, etc.

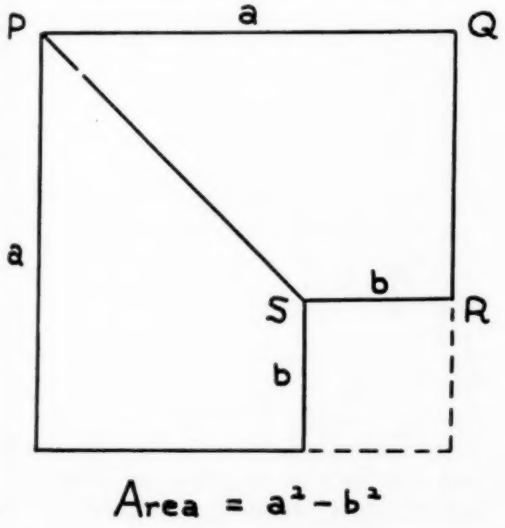


FIG. 1

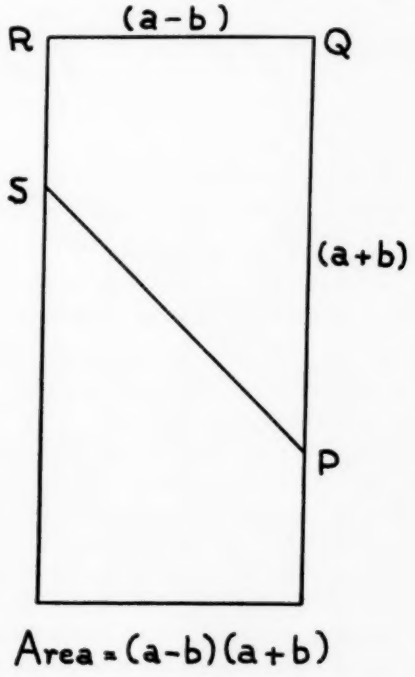


FIG. 2

* *Acknowledgement:* Drawings for publication in this section were produced by Patricia McGroder, Macalester College, St. Paul, Minnesota.

With an eight inch circle, half of red and half of yellow felt, cut into fourteen equal sectors, and two half-size sectors, the usual demonstration of the formula for area from the knowledge of the circumference is simple and expeditious. The tribulations of paper, paste and paint are avoided and a normally static demonstration becomes dynamic.

The bright colors in which these felts can be obtained make them admirably clear and easily seen. Preparation of the aids is speedy, and if each set is placed in a large labelled envelope storage is easy.

The following demonstration that $a^2 - b^2 = (a+b)(a-b)$, simpler than the usual one and more easily understood, does not seem to be widely known. The cutting of the two pieces of felt takes only a few minutes. (See Figure 1.) To complete the demonstration the trapezium $PQRS$ must be turned over and placed in the position shown in Figure 2.

Tom F. Battersby
County Mining and Technical School
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A Variable Parabola Demonstrator

A useful device for certain demonstrations concerning the parabola is illustrated in the accompanying figure. (See Figure 3.) A parabola is formed by a colored cord passed through eyelets at B_1, B_2, B_3, . . . B_n. A_1P is a rod from which the parabola is suspended. As A_1P is tilted at various angles with the horizontal the parabola changes dimensions; that is, the constants change, but the figure remains a parabola.

A wooden dowel one-half inch in diameter is ideal for the rod A_1P. Any kind of light weight string is suitable for the lines A_1B_1, A_2B_2, etc. Fish-line weights fastened to the ends of these strings will keep the parabola regular as the rod is tilted. For the eyelets at B_1, . . . B_n, one-half inch plastic rings, which can be obtained from any department store notions counter, will be found convenient. To fasten the strings at A_1, . . . A_n, holes should be drilled in the dowel, the strings pulled through, and wooden wedges (toothpicks or match stems) driven into the holes along side of the strings. The colored cord should be tied to the eyelet at B_n but simply threaded through the others so that it will slide freely when the rod is tilted. To keep the cord taut when the dowel is tilted a weight should be fastened to the cord at the left end.

The mathematics of this device is simple enough. Let A_1P and A_nB_n represent the x-axis and y-axis respectively in a rectangular coordinate system. The equation of the parabola in the position shown in Figure 3 will then be $x^2 = k(y+h)$, where k is negative. In accordance with this scheme A_1B_1, . . . A_nB_n are ordinates, and A_1A_n, A_2A_n, . . . are abscissas of the points B_1, . . . B_n.

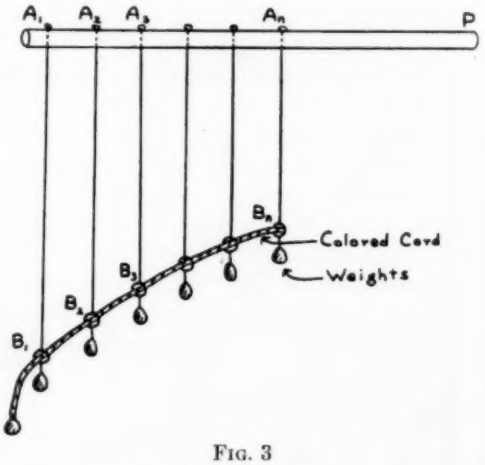

Fig. 3

In constructing the device the work can obviously be simplified by using the equation $x^2 = y$ to determine the shape of the curve, and then adding a constant length (A_nB_n) to each ordinate. It is an interesting problem for the student to show algebraically that the curve is always a parabola, no matter how the rod is tilted.

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Purdue University
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Some Practical Ideas on Making Cross Number Puzzles for Junior High School Students

Too often students become bored with grinding out answer after answer. The purpose of this article is to suggest a motivating device which will enable the teacher to re-create interest in solving mathematical problems. The device involves the construction of cross number puzzles which students can complete by writing in the correct answers to a set of problem exercises.

Directions for construction listed below and the accompanying illustration are suggestive of what can be done. The puzzle which is worked out is suitable for seventh and eighth graders. Teachers may construct puzzles themselves and use them in regular classroom situations instead of traditional tests and written exercises, or students and teachers can work them out together.

Steps to Follow

1. Select a basic square pattern to follow—6 by 6, 8 by 8, or 12 by 12.
2. Select from two to four definitions or problems per row horizontally; also follow this pattern vertically.
3. Each answer to a problem or definition should not be longer than can be written in four squares of any one row.
4. State each definition or problem very briefly. (See the sample puzzle.) When making problems for the horizontal rows very little difficulty will be found.
5. In selecting problems for the vertical rows a little clever thinking may be needed to find problems with answers that will fit the number combinations already given by the horizontal definitions and problems.
6. Select a title for each puzzle to add to its interest. The following are suggestive: "Areas and Perimeters," "Whole Number Processes," "Volumes," "Units of Measure," and "General Processes."
7. In the beginning, it is better to start with short puzzles than to plan long ones. In order to maintain student interest at a high level it is best not to include more than 25 items horizontally and the same number vertically.

The puzzles may be drawn and worked out on classroom blackboards or separate copies made for all students by duplicator processes. After presenting the idea to the class it is well to devote one full period to giving directions—i.e., showing the students how to outline the squares, and acquainting them with the numbering system. Students should be permitted to work at their own rate in class, and be encouraged to make similar puzzles at home.

Sample Puzzle: Areas and Perimeters

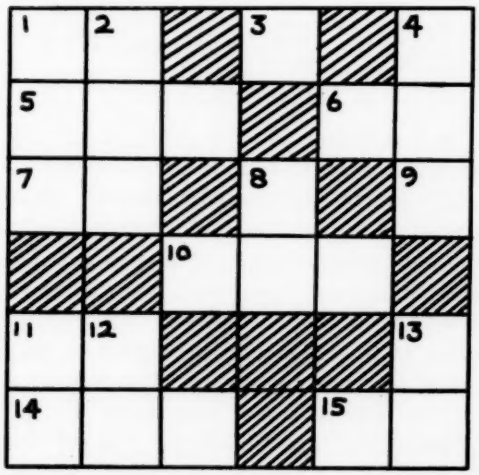

Fig. 4

Horizontal

1. Perimeter of a 9″ square.
3. Area of a 1″ square.
5. Area of a rug 9′×12′.
6. Circumference of a circle with a diameter of 7″.

7. Area of a rectangle $16'' \times 3\frac{1}{2}''$.
8. How many sides has a square?
10. Find the area of a triangle with a base of 16″ and an altitude of 14″.
11. Same definition as (11) vertical.
14. Perimeter of a wheel 49″ in diameter.
15. How many 2″ squares can be made to fit into a rectangle $4'' \times 10''$?

Vertical

1. Area of a room $30' \times 10\frac{1}{2}'$.
2. Area of a field 60 rd. $\times$ 10.1 rd.
3. Area of a unit square.
4. Area of a square 15 cm. on a side.
8. Perimeter of a fence 12 yd. $\times 8\frac{1}{2}$ yd.
11. Perimeter of a triangle which is 7″ on each side.
12. Perimeter of a rectangle 4.8 cm. $\times$ 2.7 cm.
13. Perimeter of a lawn which measures $23\frac{3}{4}$ ft. in length and $21\frac{1}{4}$ ft. in width.

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Elm School
Elmwood Park
Chicago, Illinois

A Trisection Device

A discussion of the problem of trisecting the general angle comes up perennially in plane geometry classes. That trisection is possible, even though it cannot be accomplished with straightedge and compasses, is common knowledge to most teachers of the subject, but a statement of this situation leaves curious students somewhat confused. The following device, which is an adaptation of the "tomahawk" described by Robert C. Yates in the *Eighteenth Yearbook*, can be produced with very few materials and should offer interested students some degree of satisfaction.[1] It is particularly useful for blackboard demonstrations.

Materials of construction include one coffee can top or other round flat disc approximately 5″ in diameter, one strip of wood $\frac{1}{2}'' \times \frac{1}{2}''$ exactly three times the radius of the disc in length, one round $\frac{1}{4}''$ dowel approximately six times the radius of the disc in length, and two 3/16″ stove bolts.

Figure 5 shows how the trisector is assembled. Let AB represent the $\frac{1}{2}'' \times \frac{1}{2}''$ wooden strip equal in length to three times the radius of the disc. Drill a $\frac{1}{4}''$ hole at the trisection point C. Note that AC is equal to the diameter of the disc, and CB is equal to its radius. Turn the wooden strip on its side and place it along a diameter of the disc. Drill two holes through the strip and the disc and secure the two pieces with stove bolts. Insert the dowel CD in AB at C. CD must be perpendicular to AB at C. Thus CD is tangent to the circular disc.

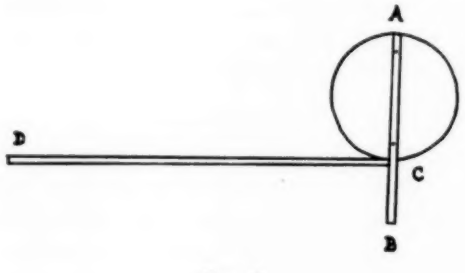

Fig. 5

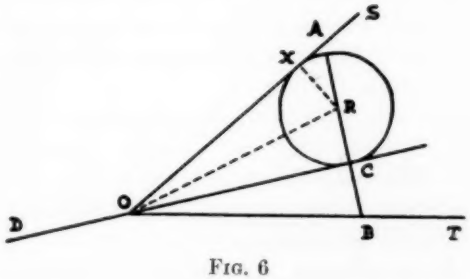

Fig. 6

Figure 6 illustrates how the device may be used to trisect an angle. To trisect angle SOT place the device so that B touches side OT. Adjust the device so that CD passes through O and the circular disc is tangent to OS. It can readily be demonstrated that right triangles XRO, ORC, and OBC are congruent. Hence angle COB is one-third of angle SOT.

[1] Yates, Robert C., "Trisection," *The National Council of Teachers of Mathematics Eighteenth Yearbook*, Bureau of Publications, Teachers College, Columbia University, 1945, New York, p. 148.

AIDS TO TEACHING

Edited by

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and

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BOOKLETS

B. 42—How to Study—How to Solve

Addison-Wesley Press, Inc., Kendall Square Building, Cambridge 42, Mass.

Booklet by H. M. Dadourian; 5″×8″; 34 pages; $.25 each.

Description: This booklet is divided into two parts. The first gives hints on how to study. It speaks of the necessity for proper mental attitude, the helpful conditions while studying alone, benefits gained from studying in groups, and the correct approach for preparing for examinations. Such topics as the following appear: incentive, interest, self-confidence, common sense, regularity in study, concentration, orderliness, thoroughness, function of the classroom, function of the instructor, making summaries before examinations, and the will to learn. The second part gives fifteen specific hints for solving problems in mathematics. After the fifteen hints are given, they are discussed singly, using illustrations from college freshman mathematics.

Appraisal: Because the maturity of tone and the types of examples slant this booklet toward college topics it will be useful to college teachers and their pupils, and to secondary school teachers. It would be useful and interesting to have such a pamphlet prepared for secondary school pupils, or to have more of this material incorporated into regular textbooks at that level. There is no doubt that teachers could spend more time analyzing aloud, with some definiteness, the principles of method which apply to mathematics problems, or to studying in general. Many have not had the opportunity to formulate for themselves the principles they use, and so this book should help them do so. The greatest danger in using such a treatment is that teacher and pupils may both fall into the trap of thinking that analysis of problems, setting them up, classifying methods, and combining formulas by rule can ever replace the free-lance thinking which knows no rules or bounds. Some weak organizers can have their scope of problems broadened by mechanical rules, but the original thinker who does the problem not covered by the former rules will still get further in mathematics.

B. 43—Algebra Applied to Monroe Formula Analysis

Booklet; 8½″×11″; 24 pages; free.

B. 44—Machine Methods for the Extraction of Square Root

Booklet; 6″×9″; 11 pages; free.

B. 45—Machine Methods for the Extraction of Cube and Other Roots

Booklet; 6″×9″; 8 pages; free.

B. 46—Application to Simple Correlation

Booklet; 5½″×8½″; 8 pages; free

B. 47—Machine Shop Engineering

Booklet; 6″×9″; 27 pages; free.

B. 48—Machine Method for Use with "A Short Method of Evaluating Determinants"

Leaflet; 6½″×9½″; 4 pages; free.

B. 43 to B. 48 available from Monroe Calculating Machine Co., Inc., Orange, New Jersey.

Description of B. 43: This is actually a simple textbook in algebra summarizing the topics which an operator of calculating

machines would need to know. Although the introduction says it is for business figuring, it is material of general algebraic interest. Some of the topics covered will show this: symbolism, signs of operation and aggregation, addition, subtraction, multiplication and division of quantities, factors, equations, axioms, transposing (how did that word get in!), fractions, exponents, etc.

Description of B. 44: Three machine methods of finding square root are described: (1) subtraction method (successive odd numbers), (2) inspection method (algorithm method), and (3) Subtraction and division method (subtracting odd numbers followed by division to get final result).

Description of B. 45: A general formula for extracting any root of a given number by calculating machine methods is presented, and then examples are worked out for third and fifth roots.

Description of B. 46: A machine method is given for computing simple correlation from a series of paired, ungrouped data which results in ΣX, ΣY, ΣX^2, ΣY^2, and ΣXY at the end of the operation. These results are all found in the dials simultaneously.

Description of B. 47: The application of computing machines to certain machine shop engineering problems is explained for the following topics: jig boring machines, odd gear ratios, the design of long and short addendum gears, and square root (by the subtraction method).

Description of B. 48: Machine methods are described for obtaining an auxiliary matrix from any given matrix. This auxiliary matrix may be used to solve the linear equations of which the elements of the given matrix are coefficients, or it may be used to evaluate the determinant of which the elements of the given matrix are elements (if the matrix is square).

Appraisal of B. 43 through B. 48: B. 43 will be useful in any algebra classes whether they are interested in machine methods or not. It is a compact summary of a great deal of elementary algebra. B. 44 and B. 45 will be interesting only to groups which have machines for practice purposes. The former could properly be used in many secondary school classes, the latter probably not. Both may be useful in college business mathematics or statistics classes. On a secondary level the possibility of using these booklets as illustrative materials should not be overlooked: they could be worked through for understanding with no intention of teaching the skills through drill for complete mastery. B. 46 applies machine methods to correlation and so will be useful on a college level for the most part. B. 47 will serve as illustrative material in all types of mathematics classes of the kind of applications which machine shops use. In allied mathematics courses which are correlated with shop work they will be of very practical use. The final booklet B. 48 is of technical importance and may serve as illustrative material in college algebra classes or practical use in business and engineering mathematics courses, but not of any use on a secondary level. All the booklets reviewed seem to be carefully written except the last leaflet which seems to depend too much on a previous article referred to on the cover. The explanation is obscure without it.

B. 49—Your Money and the Federal Reserve System

The Federal Reserve Bank, Minneapolis, Minnesota.

Booklet; 10″×12½″; 32 pages; free.

Description: This booklet tells the story of money and the federal reserve system by discussing the following topics: What is money? How does money go around? How do you get your money? Where does new currency come from? Where does old currency go? Why will money buy things? How can so little cash finance so much business? Your money and the federal reserve system. How the Federal Reserve Banks serve industry, government, banks and agriculture. The federal reserve sys-

tem as a stabilizing force. Each topic is described concisely in words and illustrated completely with photographs.

Appraisal: This attractive booklet, written in very readable language and illustrated in a fashion similar to *Life* magazine, should be obtained by every mathematics and social studies teacher. Although it contains no mathematics other than simple data, it will give the social setting for a unit on banking. The photography is excellent and shows each step in the processes described in the text. The paper is heavy and durable. The titles are printed in green and the text is printed in heavy black type.

B. 50—Facts You Should Know about Life Insurance

National Better Business Bureau, Inc., Chrysler Building, New York 17, New York.

Booklet; $5\frac{1}{2}'' \times 8\frac{1}{4}''$; 16 pages; $.05.

Description: An impartial treatment of the four fundamental types of insurance is presented in terms which are understandable for the layman. Information is given concerning the methods employed in calculating premiums and facts which must be considered if policies are to be purchased intelligently. The treatise is excellently handled.

The paper-covered booklet was not designed for prolonged use for cover and pages easily become detached and will not withstand much abuse. The print is clear and heavy type has been employed discriminately to emphasize important topics.

Appraisal: The material contained is pertinent to the new socialized junior high school mathematics curriculum and high school courses in general mathematics and consumer mathematics. This impartial treatment is superior to the sections on life insurance which are included in many mathematics texts. The booklet is excellently arranged and can do much to prepare students for making an expenditure which is a necessity rather than a luxury.

The booklet, as it stands, is appropriate for use on the high school level but may be adapted for use on the junior high school level by explaining such technical terms as "premium," "annuity," and "beneficiary."

Where school funds permit, it may be wise to purchase the booklets in quantities sufficient for each student. This expenditure is certainly justified by the results which may be easily attained. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

B. 51—Where Does the Money Go?

Union Dime Savings Bank, Avenue of the Americas, 40th St., New York 18, New York.

Booklet; $8'' \times 10\frac{1}{2}''$; 6 pages; free.

Description: The three-month expense record is designed as a guide for regular saving and wise spending. Each of the monthly records occupies two pages; seventeen columns are provided for recording expenditures made during the month. Among the expenses considered are savings, taxes, food, shelter, clothing, and advancement. Explanations appear at the heads of columns so that the budgeteer will be enabled to determine where his expenditures should be tabulated.

The booklet is printed in blue and red upon moderately thin white paper. It is not designed for durability but rather as a temporary expense account to be used only once.

Appraisal: The principal values of the expense record are in teaching the types of expenditures required and methods of wise budgeting. When used on the secondary level, the record will necessarily be somewhat artificial since most students have neither large incomes nor the expenses considered in the booklet. Nevertheless, a wise choice of money management problems by the teacher may result in real and lasting learning. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

B.52—Railroad 'Rithmetic Book I

Baltimore and Ohio Railroad Company; Baltimore, Maryland

Booklet; $8\frac{1}{2}'' \times 11''$; 60 pages; Free; Olive W. Dennis, Editor

Description: This is a booklet of railroad problems to supplement problems of the standard elementary textbooks. It contains ten units on topics such as: reading and writing numbers, using whole numbers, reading and using tables, using fractions, using decimals, finding per cent, solving two-step problems, using measurements, and using graphs. Answers are given for all problems in a separate section. The problems are real problems with data based on material supplied by many railroad companies. The problems are based on data such as miles of rail travel, cost of tickets, freight and maintenance, taxes, dimensions of cars, sharing freight charges, travel time, making change, areas, supplies, etc.

B.53—Railroad 'Rithmetic Book II

Baltimore and Ohio Railroad Company; Baltimore, Maryland

Booklet; $8\frac{1}{2}'' \times 11''$; 44 pages; 1950; Free; Olive W. Dennis, Editor

Description: This is a booklet of railroad problems to supplement problems of the standard textbook. It contains units on such topics as: percentage, passenger fares and freight fares, measurement, circle problems, cylinder problems, scale drawing, reading tables, and graphs. Answers are given for all problems in a separate section. The problems are all real problems with the data based on material supplied by many railroad companies. The problems are based on data such as the cost of supplies, amount of taxes, travel costs and data, car dimensions, volumes, sharing freight charges, distances and travel time, etc. The illustrations include time zones, scale drawings of cars, and a variety of graphs.

Appraisal of B.52 and B.53: These booklets will furnish the mathematics teacher with real life problems based on conditions encountered in the railroad industry since the statistical data and operating procedures have been obtained from railroad sources. As is so often true with real problems, the numbers involved are not always simple numbers, for example the investment return earned in 1947 was 3.41%. However, the booklet contains a great variety of problems suitable for supplementary work by the pupil with special ability or interest in railroads. The material is not copyrighted since the teacher is urged by the publisher to use it in courses of study or textbooks. It contains little direct advertising but, of course, it does contain problems that show the contribution of the railroads to society in such terms as economical transportation, payment of taxes, employment, etc. These are booklets that will furnish the elementary as well as secondary teacher of mathematics the kind of practical problems based on real data that are so often lacking in the textbook.

CHARTS

C.19—Graph Chart

The Mathaids Company; 336 Kirk Avenue; Syracuse, New York

Chart; $32'' \times 40''$; $8.50 (presswood) or $8.95 (Duron)

Description: A $32'' \times 40''$ tempered presswood board is coated with black "Liquislate"; horizontal and vertical lines are drawn at $1\frac{1}{4}''$ intervals to form a rectangular coordinate chart. The Duron model has "Liquislate" on the back and can be used as a portable blackboard. There is a 1-inch border around the top and right sides and borders of $3\frac{7}{8}''$ and $3\frac{3}{8}''$ on the left and bottom; these larger spaces are used for recording information concerning the graphs to be drawn.

Each corner of the board is reinforced with a brass jacket and metal eyelets protect the holes which facilitate hanging the

chart. Visibility of the squares is excellent due to the high degree of contrast between the "Liquislate" and bright yellow lines. The material is quite durable and the metal jackets will prolong the life of the chart. Chalk marks may be removed with an ordinary blackboard eraser but better results are obtained with the use of a clean rag.

Appraisal: This type of portable graph chart has been used extensively in classrooms which are not supplied with graphic wall blackboards. The problem of storage, however, makes this type of board less desirable than the flexible stencil-type or the blackboard chart which may be rolled up when not in use.

$8.50 or $8.95 is a high price to pay for this type of graph although it is sturdy and has several other desirable physical characteristics. (Reviewed by Bernard Singer, Hyannis, Mass.)

FILMSTRIPS

FS.74—Maps and Their Meanings

FS.75—We Live on a Huge Ball

FS.76—Flat Maps of a Round World

FS.77—Maps and Man

Popular Science Publishing Co.; 353 Fourth Avenue, New York 10, New York

35 mm filmstrips; Color and black and white; Approximately 50 frames each; Teaching Guide

Description of FS.74: This elementary filmstrip on maps begins by showing the relationship between pictures and maps. It then shows how symbols are used to indicate buildings, land and water forms, and other man-made or natural features. Examples of legends or keys for a map are illustrated. Direction and location on a map are related to east-west or north-south lines. The meaning and use of scale are illustrated in some detail. Maps using color to show relief or rainfall extend the information given by maps.

Description of FS.75: This filmstrip contains a series of pictures almost all of which are of globes. It shows the relationship between a map on a globe and a flat map. A series of drawings develops the idea of a North and South pole, equator and parallels of latitude to locate position on the earth. The relation of location and climate are presented briefly. The major emphasis of this filmstrip is the use of latitude to locate positions on the earth.

Description of FS.76: This filmstrip begins by showing the meaning of meridians of longitude on a globe. By numbering the meridians and using the latitude discussed in the previous strip of this series, the location of a city, Denver, is shown. The difficulty of making a two-dimensional map of the earth is illustrated by comparing the mercator, sinusoidal, and polar-equidistant projections. The kind of distortion involved is shown by comparing the map with the globe on the same frame. Several frames are used to give the pupil practice in using meridians and parallels to determine locations and distances with emphasis on longitude.

Description of FS.77: Maps such as weather maps, Polar Equidistant, projections, Mercator maps, maps of wind and ocean currents, maps of agricultural products, soil fertility, railroad networks, population, water resources, rainfall, and temperatures are used to illustrate the great variety and uses of maps. The convenience of a map to present a great deal of information concisely is emphasized.

Appraisal of FS.74 through FS.77: These filmstrips present an integrated elementary series on the reading of maps intended for use in the fifth, sixth, and seventh grade social studies classes. They will be useful for the mathematics teacher for background material when discussing projections and scale. However, most teachers will prefer to use real maps or a globe, to illustrate types of maps, symbols, or scale but the filmstrip is a convenient method of showing variety maps. The drawings are appropriate and clear with captions that require frequent class re-

sponses. A teaching guide which includes outcomes, generalizations, vocabulary, suggested activities and prints of every frame of the entire series will be most helpful for teachers.

INSTRUMENTS

I.28—World Hour Chart

Newsweek; 152 West 42nd Street, New York 18, New York

Chart; $3\frac{3}{4}'' \times 4\frac{1}{2}''$; Free

Description: The pocket-size chart consists of a piece of cardboard upon which are located the twenty-four time zones of the world. Two slots at the left top and bottom of the cardboard provide space for a non-ending tape upon which are printed twelve red digits and a like number in black to represent the twenty-four hours in each day. The tape may be turned until the correct Greenwich time is opposite the location of the observer. The chart is then in alignment for reading the times all over the world.

The chart is printed upon a sturdy piece of cardboard and will survive much use if care is taken to protect the fabric tape from damage.

Appraisal: The device may be used to motivate the study of time zones. With it the teacher may explain the concept of time variations due to revolution of the earth about its axis; meaning of the international date line becomes clearer when reference is made to the chart and students will enjoy checking answers to problems.

The chart is far too small to be used effectively as a demonstration device; each student should be supplied with one. (Reviewed by Bernard Singer, Hyannis, Mass.)

MODELS

M.15—Celestial Globe

Denoyer-Geppert Co.; 5235–5237 Ravenswood Ave., Chicago 40, Illinois

Globe; (G16 C 05) 16″; $40.00 (G12 C 05) 12″; $22.50

Description: The DG new 16″ celestial globe shows the important stars down to the sixth magnitude, distinctly on a light blue background. The latest constellation boundaries as fixed by the International Astronomical Union are used and the globe map is correct for Epoch 1930. It is mounted with either movable meridian with horizon (G16C5), Liberty stand with horizon (G16C05), plain stand (G16C1), or weighted base, movable meridian (G16C3).

Appraisal: If a student can pick up an object in his hands or twirl it around, he is more apt to learn the principles being put forth than is the case when all the work is done by the teacher. A globe such as this with all the essential markings would seem to be a most valuable instrument in the classroom. (Reviewed by Jacqueline LeMay)

PICTURES

P.9—Galileo

Denoyer-Geppert Company; 5235 Ravenswood Avenue, Chicago 40, Illinois

Picture; 26″×35″; Unmounted $2.00, five pictures $7.50, ten pictures $10.00; Lacquered panel, passe-partout edges $6.00; Lacquered panel, 3″ frame $10.50; Under glass, 3″ frame $12.50.

Description: Galileo is seen sitting at a desk, fingering a globe and writing. Another larger globe, a plumb-bob (possibly a sundial or groma) and an instrument which may be a telescope surround him. The composition and coloring in the picture are very attractive.

Appraisal: For static, artistic pictures of mathematicians or users of mathematics this is certainly much better than the usual head-and-shoulders presentation. The instruments, even though their use is not always clear, are suggestive of mathematical activity. With supplementary reading, such pictures would make mathematics seem to be the useful tool of adult workers, rather than a series of disconnected problems in a textbook.

A small leaflet showing this picture is available upon request.

SOURCES OF MATERIAL FOR LABORATORY WORK

SL.20—Thermium Film

Yoder Instruments; 140 North Market Street, East Palestine, Ohio

Protective Film; Several sheet sizes and roll sizes available; price varies depending on size.

Description: Thermium Film is a protective, transparent covering that can be applied to pictures, charts, drawings, records, etc. It protects them from dirt, smudging and tearing. It is applied by placing the sheet over the material to be covered and ironing over the sheet with a warm flatiron or specially designed Thermo-Welder. The film is available in a large assortment of cut sheet sizes as well as in roll form. The prices range from seventy-five cents for a dozen sheets $8\frac{1}{2}\times 11$ inches to $16.50 for a 100 foot roll of 25 inch width.

Appraisal: This protective film is completely transparent and colorless. It is easy to apply if one has the iron at the right temperature. It will be necessary for the teacher to experiment with different temperatures for the iron to determine the best temperature for effective use. (A small size Thermo-Welder for $4.25 will solve this problem.) It is an effective material to use for the protection of surfaces, even though it is expensive. Material covered by this film can be cleaned by wiping with a damp cloth. It is advertised that it will not deteriorate or discolor with age.

MATHEMATICAL RECREATIONS

Edited by AARON BAKST

School of Education, New York University, New York 3, N. Y.

MULTIPLICATION may be made comparatively easy if we follow the procedure illustrated by the Arab mathematician *Beha ed-din al'Amili, Mohamed ibn Hosein* (1547–1622). All that one needs to know is compressed into the multiplication facts limited to the first nine integers and into the proficiency in column addition of the first nine integers.

Let us illustrate this method by a few examples. We will first multiply 36 by 83.

Draw a rectangle containing four cells. Four cells are required because each of the numbers consists of two digits. If one of the numbers contains three digits and the other number contains four digits, then the required rectangle will contain twelve cells. Generally, if one number contains m digits and the other number contains n digits, then the required rectangle will contain mn cells.

The rectangle is so drawn that one of its vertices rests on a horizontal straight line as shown in Figure 1. Draw vertical diagonals of the cells of the rectangle until they meet the horizontal straight line AB.

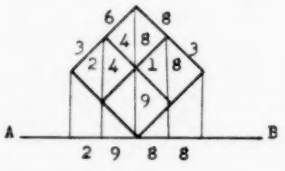

FIG. 1

Write the two numbers along the two upper sides of the rectangle is shown in Figure 1. Note that one digit is placed so that it occupies a position along the outer side of a single cell. Also note that every diagonal divides its cells into two *subcells*.

Multiply 6 by 8. The product is 48. Write this product in the cell which is ad-

jacent to 6 and 8. Furthermore, place one and only one digit in a subcell (as shown in the figure above).

Multiply 3 by 8 and write the product 24 in a manner described in the preceding paragraph. Note that the cell for this product is determined according to the principles which are employed in "tables" of numerical data. Then multiply 6 by 3 and write the product 18 in the manner described above. Finally, multiply 3 by 3. Note, however, that the product 9 is written in the subcell which is on the right of the diagonal of the cell, while the subcell on the left of the diagonal remains unfilled. The reader will surmise by this time that the subcell on the right of a diagonal is assigned to *units*, while the subcell on the left of a diagonal is assigned to *tens*.

After all the separate multiplications are completed perform column additions as indicated in the figure above. If the sum of a column is greater than 9, "carry" the units in the *tens* to the next column on the left. Thus the product of 36 and 83 is 2988.

Let us multiply 68924 and 3931. The procedure is exactly the same as described above. The required rectangle will contain 20 cells. This is shown in Figure 2.

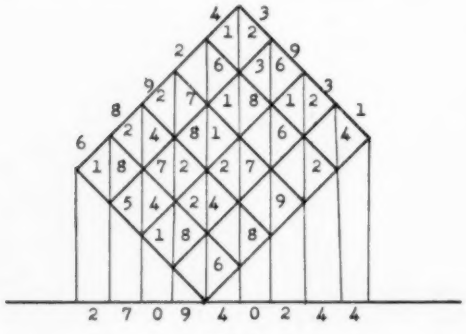

FIG. 2

The method of multiplication described and illustrated above can be theoretically developed on the level of eleventh year algebra. It may be noted that the vertical lines (the diagonals of the rectangular cells) represent the demarcation lines of the "places of significance" in the product of two numbers.

If we write the two numbers as

$$10^m a_m + 10_m^{m-1} a_{m-1} + \cdots + 10^2 a_2 + 10a_1 + a_0$$

and

$$10^n b_n + 10^{n-1} b_{n-1} + \cdots + 10^2 b_2 + 10b_1 + b_0,$$

where

$$a_i = 0, 1, 2, \cdots, 8, 9, \qquad i = 0, 1, 2, \cdots, m$$

and

$$b_j = 0, 1, 2, \cdots, 8, 9, \qquad j = 0, 1, 2, \cdots n.$$

The product of the two numbers is obtained as the product of two polynomials. Since the diagonals of the cells provide for the "places of significance" and since at no time the product of a_i and b_j can exceed 81, two subcells are sufficient for the writing of the individual products of the coefficients a_i and b_j which are found in the polynomial representations of the two numbers respectively.

Here is an advance notice of the problem which will be discussed in the next issue of THE MATHEMATICS TEACHER. It is an interesting teaser for a Geometry classroom. Divide the area of a circle into n (n, any integer) equal parts, but the division of the circumference of the circle is not permitted, nor is the drawing of concentric circles or of parallel chords permitted. (These cases will be discussed in a subsequent issue of THE MATHEMATICS TEACHER.)

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

16. Algebraic Tick-Tack-Toe

This version of an old game was described to the editor of this department by Mrs. Lloyd Middleton Garris of the Lincoln Memorial High School of Palmetto, Florida.

Two students alternately fill in with signed numbers the cells of the ordinary tick-tack-toe diagram. Their object is to make each row, column, and diagonal sum to zero. The first student to make the sum of a row, column, or diagonal something other than zero loses the round.

Figure 1 shows a typical round played by "*A*" (italicized numbers) and "*B*". The sequence of play was as follows:

−9	*−5*	14
13	5	*−18*
−4	*0*	*4*

FIG. 1

A, 4; *B*, 5; *A*, −9; *B*, −5; *A*, 0; *B*, 14; *A*, −18; *B*, 13; *A*, −4. This game was won by *B*.

The possibilities of varying the game by such devices as choosing a sum other than zero, using literal terms also, or using other operations than addition are obvious as are also the uses of the game for motivating drill and reviews and for merely adding to the fun and interest to be found in a mathematics class or club.

The connection with magic squares is also obvious and might well motivate a report or project dealing with this latter topic.

17. Geometric Proof of the Addition Formulas for the Tangent

In this note the formulas for tan $(\alpha+\beta)$ and tan $(\alpha-\beta)$ are developed geometrically. Similar developments for sin $(\alpha\pm\beta)$ and cos $(\alpha\pm\beta)$, are given in textbooks on trigonometry, while tan $(\alpha+\beta)$ and tan $(\alpha-\beta)$ are commonly developed algebraically.

For the sake of simplicity the angles α and β are restricted to positive acute angles whose sum is an acute angle. An extension of the method of this note may easily be made so as to include all angles α and β for which tan α, tan β, tan $(\alpha+\beta)$ and tan $(\alpha-\beta)$ are defined.

Let (Fig. 1) α and β be two acute angles whose sum is an acute angle. Let P be any point on the positive end of the terminal side of $(\alpha+\beta)$. Drop the perpendiculars PA and PC to the initial and terminal sides respectively of α. From C drop the perpendiculars CD and CB to the lines PA and OA. Since the sides PC and PD of $\angle CPD$ are perpendicular to the sides of α, and since α and $\angle CPD$ are both acute, they are equal. Therefore, right triangles DPC and OBC are similar.

From Fig. 1,

$$\tan(\alpha+\beta)=\frac{AP}{OA}=\frac{AD+DP}{OB-AB}$$

$$=\frac{BC+DP}{OB-DC} \qquad (1)$$

since $AD=BC$ and $AB=DC$. When both numerator and denominator of the right side of (1) are divided by OB,

$$\tan(\alpha+\beta)=\frac{\frac{BC}{OB}+\frac{DP}{OB}}{1-\frac{DC}{OB}}. \qquad (2)$$

Now

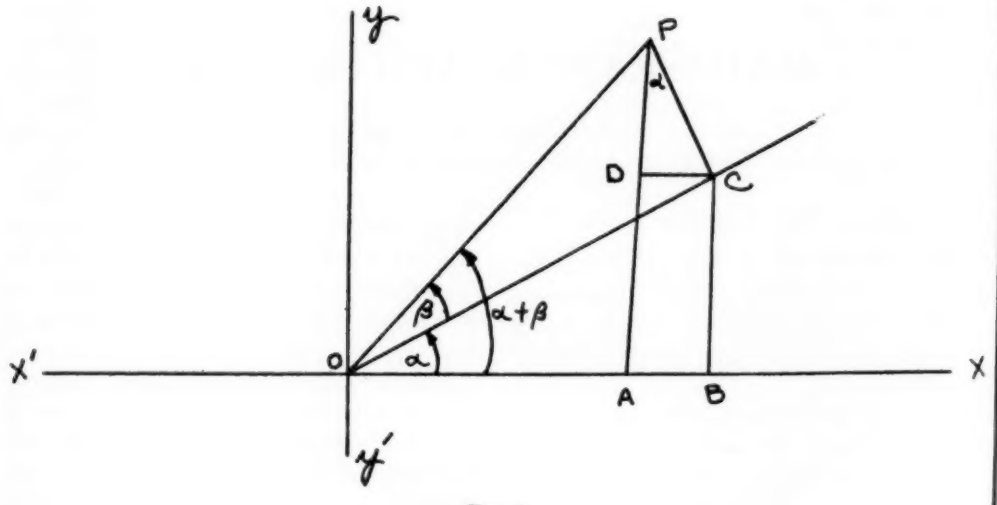

FIG. 1

$$\frac{DP}{OB}=\frac{DP}{PC}\cdot\frac{PC}{OC}\cdot\frac{OC}{OB} \quad \text{and}$$

$$\frac{DC}{OB}=\frac{DC}{PC}\cdot\frac{PC}{OC}\cdot\frac{OC}{OB}.$$

Consequently,

$$\tan(\alpha+\beta)=\frac{\frac{BC}{OB}+\frac{DP}{PC}\cdot\frac{PC}{OC}\cdot\frac{OC}{OB}}{1-\frac{DC}{PC}\cdot\frac{PC}{OC}\cdot\frac{OC}{OB}}. \quad (3)$$

From the similar triangles PDC and OBC we have

$$\frac{DP}{PC}=\frac{OB}{OC}$$

and hence

$$\frac{DP}{PC}\cdot\frac{OC}{OB}=1.$$

Also, from the same triangles

$$\frac{OC}{OB}=\frac{PC}{PD}$$

and consequently

$$\frac{DC}{PC}\cdot\frac{OC}{OB}=\frac{DC}{PC}\cdot\frac{PC}{PD}=\frac{DC}{PD}.$$

Then (3) becomes

$$\tan(\alpha+\beta)=\frac{\frac{BC}{OB}+\frac{PC}{OC}}{1-\frac{DC}{PD}\cdot\frac{PC}{OC}}. \quad (4)$$

Since

$$\frac{BC}{OB}=\tan\alpha, \quad \frac{DC}{PD}=\tan\alpha,$$

and

$$\frac{PC}{OC}=\tan\beta,$$

the substitution of these values in (4) yields

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}. \quad (5)$$

To prove the formula for $\tan(\alpha-\beta)$ take α and β acute, and $\alpha>\beta$ so that $\alpha-\beta$ is a positive acute angle. This restriction is made only for simplicity. Let (Fig. 2) P be a point on the terminal side of $\alpha-\beta$. From P drop perpendiculars PC and PA to the terminal and initial sides respectively of α. From C drop a perpendicular CB to OA and from P drop a perpendicular PD to CB. From Fig. 2,

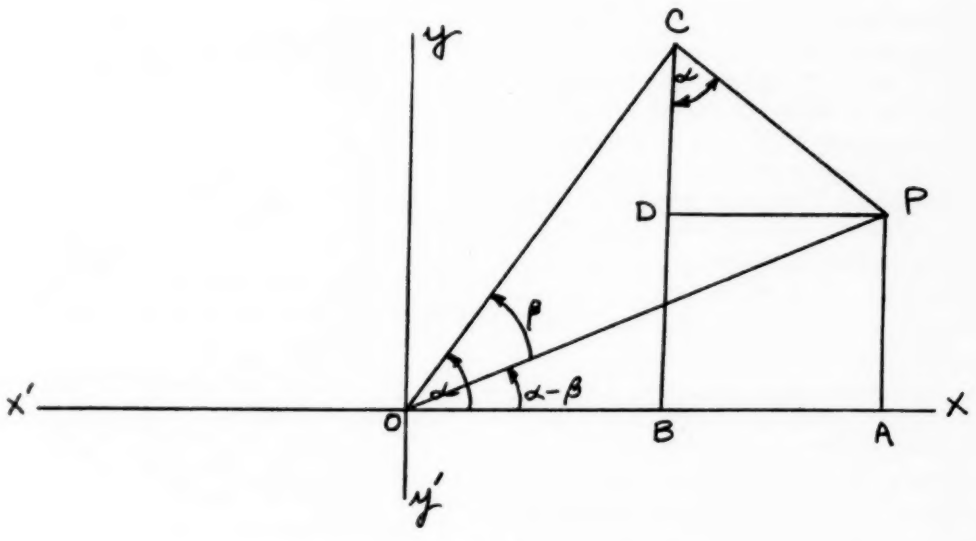

FIG. 2

$$\tan(\alpha-\beta)=\frac{AP}{OA}=\frac{BD}{OB+BA}$$
$$=\frac{BC-DC}{OB+DP}, \qquad (6)$$

since $AP=BD$ and $BA=DP$. If the numerator and the denominator of the right side of (6) are divided by OB,

$$\tan(\alpha-\beta)=\frac{\frac{BC}{OB}-\frac{DC}{OB}}{1+\frac{DP}{OB}}, \qquad (7)$$

which may be written

$$\tan(\alpha-\beta)=\frac{\frac{BC}{OB}-\frac{DC}{CP}\cdot\frac{CP}{OC}\cdot\frac{OC}{OB}}{1+\frac{DP}{CP}\cdot\frac{CP}{OC}\cdot\frac{OC}{OB}}. \qquad (8)$$

From the similar triangles OBC and CDP,

$$\frac{DC}{CP}=\frac{OB}{OC}$$

and hence

$$\frac{DC}{CP}\cdot\frac{OC}{OB}=1.$$

From the same triangles

$$\frac{OC}{OB}=\frac{CP}{CD}$$

and

$$\frac{DP}{CP}\cdot\frac{OC}{OB}=\frac{DP}{CP}\cdot\frac{CP}{CD}=\frac{DP}{CD}.$$

Equation (8) may now be written

$$\tan(\alpha-\beta)=\frac{\frac{BC}{OB}-\frac{CP}{OC}\cdot\frac{DC}{CP}\cdot\frac{OC}{OB}}{1+\frac{DP}{CP}\cdot\frac{OC}{OB}\cdot\frac{CP}{OC}}$$
$$=\frac{\frac{BC}{OB}-\frac{CP}{OC}}{1+\frac{DP}{CD}\cdot\frac{CP}{OC}}. \qquad (9)$$

Now

$$\frac{BC}{OB}=\tan\alpha,\ \frac{DP}{CD}=\tan\alpha,\ \text{and}\ \frac{CP}{OC}=\tan\beta.$$

Hence (9) becomes

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}, \qquad (10)$$

which is the required formula.

By means of Figures 1 and 2 and the above methods, the formulas for cot $(\alpha+\beta)$ and cot $(\alpha-\beta)$ may be derived. Similarly formulas for sec $(\alpha\pm\beta)$ and csc $(\alpha\pm\beta)$, in terms of csc α, csc β, sec α, and sec β, which are not in general use, may be obtained. Thus it is possible to derive geometrically addition formulas for the six trigonometric functions.

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18. The Nedians of a Plane Triangle

The *medians* of a triangle are the straight lines drawn from the angular points to the mid-points of the opposite sides. It is well known that the medians are concurrent, the point of intersection being the centroid of the triangular area. Fig. 2 illustrates this. Denoting the lengths of the sides of the triangle by a, b, c, and the lengths of the medians AD, BE, CF

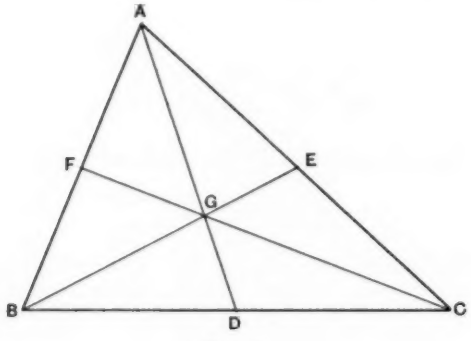

Fig. 2

by m_1, m_2, m_3, respectively it is easily proved that

(1) $\quad m_1^2+m_2^2+m_3^2=(3/4)(a^2+b^2+c^2).$

If the sides BC, CA, AB are divided at D', E', F', respectively (Fig. 3), such that

$$BD'=\frac{1}{N}BC,\quad CE'=\frac{1}{N}CA,\quad AF'=\frac{1}{N}AB$$

(where N is a positive integer) the lines AD', BE', CF', may be called the *nedians* (just as medians refer to the mid-points, nedians refer to the $1/N$ points).

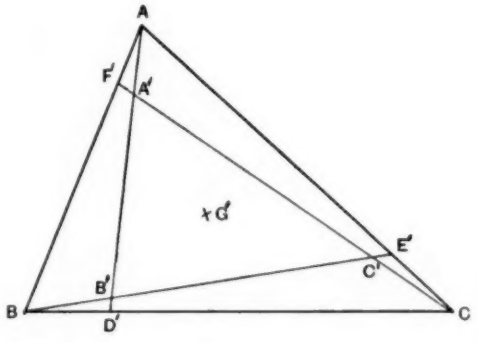

Fig. 3

Denoting the lengths of the nedians by n_1, n_2, n_3, respectively, it may be shown by analytical geometry that

$$n_1^2+n_2^2+n_3^2 = \frac{(N^2-N+1)}{N^2}(a^2+b^2+c^2). \tag{2}$$

Putting $N=2$, makes the nedians become the medians and the value of

$$\frac{(N^2-N+1)}{N^2}$$

becomes $\frac{3}{4}$, as in (1).

Putting $N=1$ places D', E', F' at C, A, B, respectively. Letting N increase without bound makes D', E', F' approach B, C, A respectively as limiting positions. In these two cases the nedians coincide with the sides of the triangle ABC and

$$\lim_{N\to\infty}\frac{(N^2-N+1)}{N}=1.$$

Calculus shows that

$$\frac{(N^2-N+1)}{N^2}$$

has a minimum value of $\frac{3}{4}$, which occurs when $N=2$, when the nedians become the medians.

The triangle $A'B'C'$ formed by the nedians may be called the Nedian Triangle. My colleague, Professor Petrie of the Ontario College of Education, proved by analytical geometry, that the area of the Nedian Triangle

(3) $= \dfrac{(N-2)^2}{(N^2+N-1)} \cdot$ (Area of the triangle ABC.)

The expression

$$\frac{(N-2)^2}{(N^2+N-1)}$$

equals unity when $N=1$, and also approaches 1 as a limit when N increases without bound. This expression has a minimum value of zero when $N=2$.

So far we have put $BD'=(1/N)BC$ and so on and gone around the triangle ABC anti-clockwise. We may also go around the triangle clockwise, making $CD''=(1/N)CB$, $BF''=(1/N)BA$ and $AE''=(1/N)AC$, (Fig. 4). We get a different set of nedians and for the sake of distinction we may call the first set the *forward nedians* and the second set the *backward nedians*, and the triangle $A'B'C'$ the *forward nedian triangle* and the

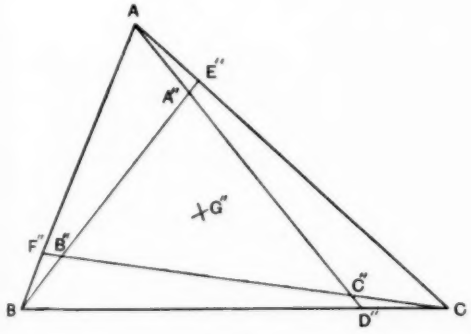

FIG. 4

triangle $A''B''C''$ the *backward nedian triangle*. Denoting the lengths of the backward nedians by n_1', n_2', n_3' respectively, we have naturally (by the same proof as for (2) and (3))

(4) $\quad n_1^2+n_2^2+n_3^2=n_1'^2+n_2'^2+n_3'^2$

and the

(5) Area of $A'B'C'$ = Area of $A''B''C''$.

The triangles $A'B'C'$, $A''B''C''$ although equal in area are not similarly-shaped. It is however, very interesting to find that their *centroids are common with the centroid of the main triangle*. The proof is easy.

If we divide each of the sides of the triangle ABC into N equal parts, and draw lines joining the vertices to the division points we get a figure like Fig. 5, which shows the case for $N=5$. The lines other than the forward and backward nedians defined as above need no special names, unless *intermediate nedians* be applied to them. It will be seen that the intersections of what may now be called corresponding pairs of forward and backward nedians lie upon the medians of the triangle ABC. That this should be so follows from *Ceva's Theorem* which states that "If three

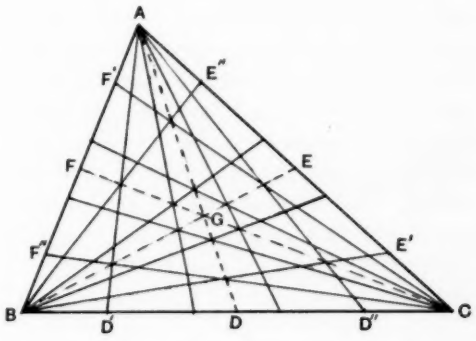

FIG. 5

concurrent straight lines are drawn from the angular points of a triangle to meet the opposite sides (Fig. 6) the product of these alternate segments taken in order is

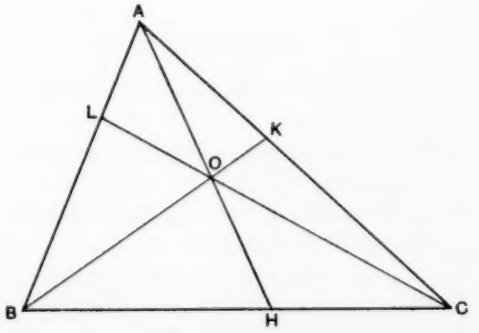

FIG. 6

equal to the product of the other three segments i.e. $BH \cdot CK \cdot AL = HC \cdot KA \cdot LB$. The converse to Ceva's Theorem is also true. These conditions are fulfilled in our array of nedians and medians, hence the prolific concurrency as illustrated in Fig. 5.

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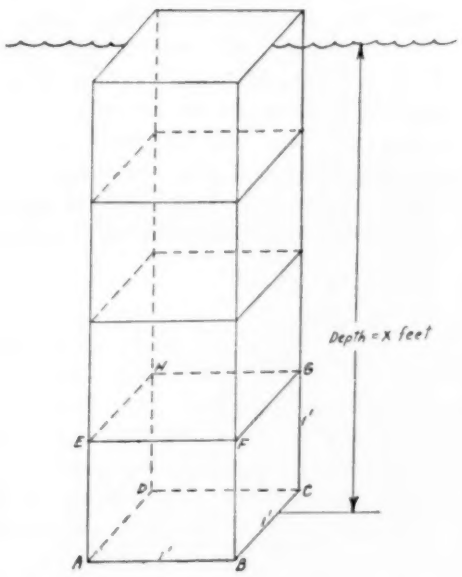
APPLICATIONS

Edited by SHELDON S. MYERS
University School, Ohio State University School, Columbus, Ohio

P.1 Gr. 10–12 USING THE METHOD OF FORMAL PROOF TO ESTABLISH TWO IMPORTANT PRINCIPLES OF PHYSICS

There are many who hold the erroneous belief that applications of high school mathematics to the sciences is impractical. This stems usually from the false assumption that too much time is needed to provide students with the scientific background necessary to understand the application. The applications of this department, and especially the one below, should prove convincingly that this need not be the case.

PRINCIPLE 1

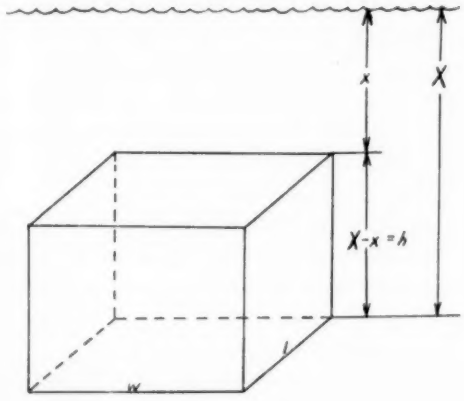


Given Data: Square foot $ABCD$ at depth of X feet in water
$EFGH-ABCD$ is a cubic foot
D is the density of water in pounds per cubic foot

Prove: Pressure, P, of water on $ABCD$ at x feet depth is xD pounds per square foot
or $P=xD$

Arguments	Authorities
1. Cube $EFGH-ABCD$ weighs D pounds.	1. Density (D) is the weight in pounds of a cubic foot. (Definition)
2. There are x cubic feet of water resting on $ABCD$.	2. Given that the depth of $ABCD$ is x feet and $ABCD$ is a square foot.
3. Total weight of the column of water on $ABCD$ is xD pounds.	3. Number of cubic feet times the weight of each cubic foot equals the total weight.
4. Pressure of $ABCD$ at x feet is xD pounds. $P=xD$	4. Pressure is the force in pounds on a square foot. (Definition)

PRINCIPLE 2



Given Data: Rectangular solid with dimensions l, w, h
Depth of top surface is x
Depth of bottom surface is X

Prove: The buoyancy of water on the rectangular solid in the figure is equal to the weight of the water displaced by the object.

Arguments	Authorities
1. Pressure, at x feet is xD lbs. per square foot. Pressure, at X feet is XD lbs. per square foot.	1. Principle 1 above: $P=xD$ Principle 1 above: $P=xD$
2. Area of top surface is lw Area of bottom surface is lw	2. Area of rectangle $=lw$
3. Force down on top surface $=lwxD$ Force up on bottom surface $=lwXD$	3. Total force = force on each square foot times the number of square feet. Also: pressure is exerted in all directions. (Postulate about liquids)
4. Buoyancy = Force up − Force down	4. Buoyancy is defined as the net upward push of the liquid on an immersed object.
5. Buoyancy $=lwXD-lwxD$	5. Substitution
6. Buoyancy $=lwD\ (X-x)$	6. Factoring
7. Buoyancy $=lwDh$	7. Substitution (see figure)
8. $lwh=V$ (Volume of solid)	8. The object is a rectangular solid.
9. Buoyancy $=DV$	9. Substitution
10. Volume of solid = volume of displaced water	10. An object displaces its own volume of liquid. (Postulate about liquids)
11. DV = weight of displaced water	11. Pounds per cubic foot (D) times number of cubic feet (V) equals total weight.
12. Buoyancy = weight of water displaced	12. Substitution

Principle 2 is known as *Archimedes Principle* in a restricted sense. Archimedes Principle applies to *any* shaped object immersed in *any* position in *any* liquid. Significant questions can be raised with the students about the nature of proof in this situation: To what extent can the principle proven above be generalized without further proof to the broader principle of Archimedes? What additional points must be established before complete generalization is possible?

AR. 5 GR. 7–9 FAMOUS EXPLOSIONS ON THE EARTH

Here is a guide sheet developed by Dr. John Kinsella at the University School, Ohio State University, in 1943.

1. Mt. Etna once blew off 450,000,000 gallons of water as steam in two weeks. How many gallons were blown off in one hour? _____

2. The Valley of Ten Thousand Smokes in Alaska, in one year, sent up 300,000 tons of a gas, smelling like rotten eggs, called hydrogen sulphide. It also gave off about four times as much hydrochloric acid gas. (This acid is used to clean metals to be soldered together). This is _____ tons.

3. In June 1912, the Alaskan volcano Katmai exploded and hurled a cubic mile of ash and fumes into the air. What is a cubic mile? _____

_____

One hundred miles away, darkness occurred for about 3 days. Ashes may settle from 6 to 9 times that distance, or _____ to _____ miles.

4. In August 1883 near Sumatra, the volcanic island, Krakatao, exploded, blowing away two-thirds of the island. A cubic mile of rock dust rose 20 miles above sea level, traveling around the earth near the equator, making dusky sunsets for months. This explosion, heard 2,000 miles away, caused tidal waves that drowned 36,000 people.

a) From New York to San Francisco is about 3,500 miles. What fractional part of this distance was the explosion heard?_____. Reduce this fraction _____.

b) How many feet is 20 miles?_____.

5. In 1815 Tomboro, a volcano in the island of Sumbawa, near Java, blew away its top. This top was a cone about 4,224 feet high and 5 miles across at its base. What was its volume in cubic miles? _____ What fractional part of a mile is 4,224 feet? _____ Reduce this fraction _____.

6. Tomboro produced about 30 to 50 cubic miles of dust and coarser materials. This dust filled the upper air, greatly reducing the intensity of the sunshine and lowering the temperature of the earth. This caused the year 1816 to be abnormally cold which is why it is called the year without a summer.

a) If it had produced 8 cubic miles of dust, this dust could have been contained in a cube _____ miles by _____ miles by _____ miles in size.

b) A cube containing 27 cubic miles would be _____ miles by _____ miles by _____ miles.

c) A cube containing 64 cubic miles would be _____ miles long, _____ miles wide and _____ miles high.

d) Therefore 30 to 50 cubic miles of dust would be contained in a cube about _____ to _____ miles long, from _____ miles to _____ miles wide and from _____ to _____ miles high.

AL. 1.5 GR. 9–10 A SIMPLE FORMULA

$I = .000012\ L(T_2 - T_1)$

I = Increase in length of paving (cement) in feet due to heat expansion.

T_1 = normal temperature of the highway in degrees Fahrenheit

T_2 = higher temperature due to summer heat

L = length in feet of concrete highway section

Problems to work: Find I when $L = 30'$, $T_2 = 110°$, $T_1 = 60°$. How are concrete highways built to prevent cracking due to expansion? What does $(T_2 - T_1)$ mean?

The above example provides the opportunity to teach the increment notation, ΔT, to stand for $T_2 - T_1$.

C. Al. 2 Gr. 10–12 Second Degree Curves and Bridges

Here are two problems used in advanced algebra and senior mathematics by Eugene Smith, University School, Ohio State University.

A concrete overhead, parabolic in shape and 20 feet high, is to span a highway 16 feet wide. The problem is to determine how high above the road it should be for each of the 16 feet. Since the parabola is a symmetric curve, it will be necessary to compute these heights for only eight one-foot intervals measured from the middle of the road to the shoulders. The formula to be used is:

$$Y = -\frac{4H}{w^2}x^2 + H$$

where H is 20 feet, w is 16 feet, and x is the distance in feet measured from the center of the road. Y is the corresponding height in feet. On graph paper draw to scale a plan of the overhead with values of Y measured to the nearest tenth of a foot.

Bridges over small rivers are occasionally elliptic in outline because of their beauty. Suppose you have to design and plan to scale an elliptic bridge with a span of 800 feet and a maximum above-water height of 40 feet. The formula to be used is:

$$Y = \frac{h}{S}\sqrt{S^2 - 4x^2}$$

where h is the maximum height above the water, S is the span, x is the distance from the middle of the river, and Y is the height in feet above the water for the corresponding value of x. Here again symmetry will make it necessary to compute only half of the values of Y. Choose your own interval so that your bridge design is possible without too much guesswork.

Here are the solutions to last month's problems:

C. Al. 1 Gr. 10–12 Arithmetic Series in the Home

(See last month's figure.) $a = 99''$, $n = 17''$, $d = 5.8''$

$$L = a - (n-1)d$$

$$L = 99 - (17-1)5.8$$

$$L = 6.2''$$

$$s = n\left(\frac{a+1}{2}\right)$$

$$s = 17\left(\frac{99+6.2}{2}\right)$$

$s = 894.2''$ or $74.5'$ A total of 74.5 feet of lath was needed to protect the cellar steps.

Al. 3 Gr. 9–10 Developing a Formula for Grading a Standardized Test of Fifty Items

Score = Rights − Wrongs $(S = R - W)$

$W = 50 - (R + 0)$

Substituting: $S = R - [50 - (R + 0)]$

Simplifying: $S = 2R + 0 - 50$

Note on Ar. 4 Gr. 7–9 Snow on the Roof

A mathematics instructor in the public schools who is also a federal weather forecaster in Springfield, Ohio came up with these figures concerning the 23.9-inch snowfall last November. Based on a city area of 8,395 acres, an estimated 3,327,744,420 pounds, or 1,663,872 tons of snow fell on the city in four days. Figuring 480 snowflakes per ounce of snow, roughly 25½ trillion flakes fluttered down, but the count is not guaranteed. When melted down, this much snow amounts to 391,762,500 gallons, or about 27 days of normal pumpage by the city waterworks.

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
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The Question: Does success with the formal steps of problem analysis in arithmetic tests insure success in solving whole problems?

The Study: Burch, Robert L. *An Evaluation of Analytic Testing in Arithmetic Problem Solving.* Duke University. 1949.

In our efforts to make students more successful in solving arithmetic problems, many of us, including authors of textbooks, have encouraged pupils to respond to such questions as, "What does the problem tell?", "What is to be found?", "What is to be done?" and "What is the best guess?". The study of Dr. Burch throws some doubt on the over-all effectiveness of this step-by-step approach.

Twenty one teachers in twenty classrooms and four schools were involved in the experiment over a period of two weeks in grades 4, 5 and 6. Two problem tests were devised, each consisting of thirteen problems whose solution depended on one arithmetic operation. Two problems involved addition, two subtraction, four multiplication and five division. The first test (S), of a multiple-choice type, required the student to select, for each problem, the correct response to each of the four questions listed in the preceding paragraph. The second test (O) consisted of the same 13 problems but required only that the problem be solved in the space provided.

Four experimental groups of classes were formed. Each group took one of the two tests during the first week of March, 1948, and the other two weeks later. The four groups were designated (O,O), (O,S), (S,S) and (S,O); the order of the letters indicates the order of taking the two tests. In addition to this testing interviews were held with 23 pupils of grade 4, 17 in grade 5 and 22 in grade 6. The purpose of these was to find out what means were used in solving the problems.

Findings

1. Success in selecting the correct process in solving the problems was usually "not dependent upon prior success with any certain step or combination of steps." (P. 121.)

2. Attempting to follow the step-by step procedure in problem solving "may actually lead to a lower level of performance on the total task." (P. 123.) In other words, the use of "steps" frequently causes confusion.

3. Furthermore, a large number of pupils who had little success in responding to the "step" questions (low S− Scores) were frequently successful in solving whole problems (high O− scores).

4. Students usually respond correctly to the question, "What is to be done?" after they react accurately to a combination of the other steps. "However, the correct response to the combination is probably not the factor which accounts for the correct determination of the algorism." This is supported by the results of the oral interviews which showed that the students did not use the formal step analysis in getting their solutions but, instead, used various cue words and phrases and capitalized on the elimination of the less plausible responses.

In concluding Dr. Burch observed that "when tests are constructed to measure separately the parts of a total task, the test builder must try to find whether performance in response to the parts is comparable to performance for the whole." (Pp. 125–6.)

The Question: What difference should philosophies of education make to the teacher of mathematics?

The Study: Ibrahim, Abdel-Aziz E. *Philosophies of Education, Their Implications for Mathematics Curricula and Classroom Procedures.* Ph.D. dissert. Ohio State University. Columbus, Ohio, 1948.

"One cannot teach successfully unless he studies the fundamental reasons for things. Unless practice stems from theory and returns to illuminate it, practice becomes in danger of being stagnant, and theory in danger of becoming futile" (p. 11). With this point of view Dr. Ibrahim proposed "to make explicit the implications for the teaching of mathematics in general education of each of the major educational philosophies and their associated theories of learning" (p. 7).

Idealism, realism and pragmatism were selected as the "major educational philosophies." Next, the essential ideas in each were explained and differences noted. Corresponding to each philosophy Dr. Ibrahim found that a particular psychology of learning was more appropriate than any other. For instance, by a deductive process he discovered that organismic psychology was more consistent with the definitions and assumptions in pragmatism than with those of idealism and realism. Finally, each of the three philosophies with their corresponding psychologies of learning were applied to mathematics teaching and curriculum problems.

Illustrations of the kinds of observations Dr. Ibrahim was led to make follow:

Teachers who believe that mathematics is a man-made creation will not guide learning in the same way as those who consider mathematical concepts, principles and propositions to be eternal truths.

Teachers who believe mathematics in its creation and development to be independent of the world of natural phenomena will not instruct in the same way as those who believe in the continuity of the abstract and concrete.

Teachers who believe that mathematics is made only by means of a hypothetico-deductive logic will not exhibit the same classroom procedures as those who see a place for intuition, induction, imagination and experimentation in the birth and development of the "queen of the sciences."

Mathematics educators who believe that a general mathematics program can be obtained by an analysis of "subjects" or of adult, life activities will not advocate the same learning experiences as those who take their cues from the present experiences and needs of youth.

In the terminal part of his study Dr. Ibrahim, aided by the assumption that the democratic ideal is the prime value in American education, emerged with a preference for pragmatism as a philosophy and organismic psychology as a theory of learning.

The study convinced this reviewer that mathematics teaching and curriculum making are honeycombed with contradictory ideas and practices despite the fact that mathematics itself is married to consistency.

Annotated References to Research Studies

Loflin, Zachariah L. *Mathematics in Chemical Engineering.* Ph.D. dissert. Teachers College, Columbia University. New York. 1949.

Only the simplest types of problems from algebra, analytic geometry, trigonometry and the calculus appeared. Algebra was the most widely used subject. The use of approximations and graphic methods, and the ability to read problems and frame equations were frequently needed.

This study should be of help in guidance and in the construction of mathematics curriculums.

TOPICS OF INTEREST TO MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF

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Mathematics and Engineering

ABOUT a year ago, writing in this magazine on the preparation of mathematics teachers, Professor Snader proposed an admirable nine-point program. One of his theses was "that the correlation of mathematics with related fields such as physical science, engineering, aviation, economics, manual arts, statistics, etc., requires teachers to be prepared through survey courses, directed professional reading and independent study." With this thesis, as with all his others, this Department is in thorough accord. If teachers are to become familiar with fields related to mathematics, fields in which they usually have had no special training, they must do it on their own or in conjunction with their professional training. In either case, a guide to the literature should prove helpful.

These bibliographic notes may also be of interest to those concerned with the mathematical training of engineers. As engineering advances, so do the demands upon the mathematical knowledge of engineers. The day has passed when a competently trained engineer could get by on a year of calculus and a half-year of differential equations. A considerable number of advanced mathematical concepts and procedures have already proved their usefulness in several engineering fields. Among these would be included: Fourier series, harmonic analysis, Gamma function, boundary-value problems, integral equations, contour integration, Fresnel integrals, Bessel functions, Legendre functions, Laplace's equations, vector analysis, matrices, the calculus of variations, the operational calculus. While not many of these subjects are now, generally speaking, part of the mathematical equipment of the average engineering graduate, more of them doubtless soon will be.

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BOOK SECTION

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THIS section presents the latest books which have been received for review in THE MATHEMATICS TEACHER. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service is available whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to THE MATHEMATICS TEACHER, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

ELEMENTARY SCHOOL

1. Texts

Arithmetic 7: The World of Numbers, by Dale Carpenter, Supervisor, Mathematics Education Section, Los Angeles City School Districts; and Elizabeth Cuthbertson, Teacher of Mathematics, Los Angeles. Cloth, 332 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. $1.68.

Arithmetic 8: The World of Numbers, by Dale Carpenter, Supervisor, Mathematics Education Section, Los Angeles City School Districts; and George F. Drake, Jr., Teacher of Mathematics, Los Angeles. Cloth, 372 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. $1.68.

Everyday Arithmetic, Junior Book 1, by Harl R. Douglass, College of Education, University of Colorado; Lucien B. Kinney, Stanford University; and Donald W. Lentz, Ridge Road School, Parma, Ohio. Cloth, ix+488 pages, 1950. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $1.88.

Everyday Arithmetic, Junior Book 2, by Harl R. Douglass, College of Education, University of Colorado; Lucien B. Kinney, Stanford University; and Donald W. Lentz, Ridge Road School, Parma, Ohio. Cloth, ix+502 pages, 1950. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $1.92.

HIGH SCHOOL

1. Algebra

Algebra: Its Big Ideas and Basic Skills, by Daymond J. Aiken, Lockport Township High School; and Kenneth B. Henderson, University of Illinois. Cloth, xv+409 pages, 1950. Harper and Brothers, 49 East 33rd Street, New York 16, N. Y. $2.48.

2. Special Courses in Mathematics

A Course in the Slide Rule and Logarithms, by E. Justin Hills, Los Angeles City Schools. Paper, iv+108 pages, 1950. Ginn and Company, Statler Building, Boston 17, Mass. $1.40.

A Manual for the Slide Rule, by Paul E. Machovina, Department of Engineering Drawing, Ohio State University. Paper, 78 pages, 1950. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $0.75.

COLLEGE

1. 1st Year Mathematics

Introduction to Algebraic Geometry, by W. Gordon Welchman, Massachusetts Institute of Technology, formerly Fellow of Sidney Sussex College, Cambridge. Cloth, x+349 pages, 1950. Cambridge University Press, 51 Madison Avenue, New York 10, N. Y. $4.50.

2. Trigonometry

Plane Trigonometry, With Tables, by H. M. Dadourian, Seabury Professor of Mathematics and Natural Philosophy (Emeritus), Trinity College. Cloth, x+109+84 pages, 1950. Addison-Wesley Press, Inc., Kendall Square, Cambridge 42, Mass. $3.50.

3. Analytical Geometry

Elementary Analytical Conics, Second Edition, by J. H. Shackleton Bailey, School House, Lancaster, England. Cloth, 378 pages, 1950. Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. $1.75.

4. Advanced Mathematics

Differential Equations, by J. E. Powell and C. P. Wells, Michigan State College. Cloth, vi+205 pages, 1950. Ginn and Company, Statler Building, Boston 17, Mass. $3.00.

Classical Mechanics, by Herbert Goldstein, Harvard University. Cloth, xii+399 pages, 1950. Addison-Wesley Press, Inc. Kendall Square, Cambridge 42, Mass. $7.00.

Lectures on Classical Differential Geometry, by Dirk J. Struik, Massachusetts Institute of Technology. Cloth, viii+221 pages, 1950. Addison-Wesley Press, Inc., Kendall Square, Cambridge 42, Mass. $6.00.

Mathematics of Relativity, by G. Y. Rainich, University of Michigan. Cloth, vii+173 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $3.50.

MISCELLANEOUS

Theory of Mental Tests, by Harold Gulliksen, Princeton University, Research Adviser, Educational Testing Service, Princeton, N. J. Cloth, xix+486 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. $6.00.

Anuario Estatistico do Brasil Ano X—1949. Instituto Brasileiro de Geografia e Estatistica. Paper, 775 pages, 1950. Servico Grafico do

Instituto Brasileiro de Geografia e Estatistica, Rio de Janeiro. (In Portuguese)

REVIEWS

Arithmetic 3: The World of Numbers, Dale Carpenter and Esther J. Swenson; *Arithmetic 4: The World of Numbers*, Dale Carpenter and Lester G. Anderson. New York. The Macmillan Co., 1950. Each iii+316 pages. $1.68.

The World of Numbers is a series of six books, grades three through eight.

The authors have been truly resourceful in capitalizing upon the daily contacts of children. They have used many and varied experiences as subject matter out of which a knowledge of arithmetic must grow.

Addition and subtraction facts are shown in attractive colored pictures of fruits, flowers, dolls, marbles, children, and other things familiar to the child in every day life. To introduce multiplication, Alice is pictured setting the table, using one knife, one fork, and two spoons at each of four places. Division is introduced in a candy store where candy canes are two cents each. Mary places her eight cents on the counter in twos and Bob places a cane near each two pennies.

After each new fact is presented there are story problems under headings that call for class discussion, and each group of problems stresses some particular activity in the child's daily living. There is much opportunity for class activity in building the various number facts.

Throughout the book the idea of interrelationships is stressed, emphasizing the fact that arithmetic is an organized system of related ideas. They are developed on the idea of making arithmetic meaningful to the learner.

Arithmetic Three and *Arithmetic Four* of this series are recommended to furnish pleasant study and practice material for meeting the specific aims and objectives of the "new arithmetic."—LORENA HOLDER, Dallas Public Schools, Dallas, Texas.

Practical Mathematics. Part I, Arithmetic With Applications (Fifth Edition), Claude Irwin Palmer and Samuel Fletcher Bibb. New York, McGraw-Hill Book Co., 1949. xii+179 pages. $1.80.

This book provides for adequate drill in the fundamental processes of arithmetic and includes an ample supply of practical problems selected from technical fields. A theoretical presentation of each topic combined with hints and illustrative examples makes the book adaptable for general use. Historical notes and information concerning simple mechanics add interest.—MARIE S. WILCOX, George Washington High School, Indianapolis, Indiana.

Mathematics in Daily Use (Revised edition), Walter Hart, Cottell Gregory, and Veryl Schult. Boston, D. C. Heath and Co., 1950. vii+376 pages. $2.04.

This text is specially designed by its authors to provide a course to meet the needs of those pupils for whom algebra will not be a profitable study or for those who will study algebra later. The authors point out that the book's "primary purposes are to produce appreciation of the presence of quantitative problems in daily life and to develop the understandings and skills that are needed to solve them."

One interesting chapter on leisure time problems includes puzzles, games of chance, and problems concerned with other leisure time activities such as knitting and crocheting. Other chapters on community activities and safeguarding family income provide pertinent problems regarding education and recreation; city, state, and federal expenses and income; and the distribution of individual incomes.

Various forms of motivation are employed in teaching the fundamental operations and their application to problems of home and school, and each of the ten chapters is followed by a review and test. The last twenty-four pages are devoted to "Tests Classified by Chapters," "Tests on Separate Processes," tables of measure, and a good index. In general, the many varied and interesting problems closely related to the experiences in the daily lives of the pupils, the attractive drawings and diagrams, and the excellent physical features of the book should assure the accomplishment of the purposes of its authors.—SADIE E. CLAPPER, Springfield High School, Springfield, Illinois.

Plane Geometry, Walter W. Hart. Boston, D. C. Heath and Co., 1950. x+382 pages. $2.00.

This textbook for tenth grade geometry classes is not just a revision of earlier textbooks by Mr. Hart; it is a new book with a new order of theorems. It would seem to be the culmination of much experience in teaching high school pupils and in writing textbooks. It is written with the normal class situation in mind—providing the minimum essentials for the slower learner, while giving sufficient material to interest and challenge the pupil of superior ability.

The introduction gives background and historical information and is followed by the first chapter which reviews geometry studied in previous courses. This makes a good "bridge" for the gap that is usually felt between previous mathematics courses and tenth grade geometry. Special attention is given to the development of scientific vocabulary.

Logical proof is developed and pupils are given many opportunities to prove "originals." Sets of supplementary exercises provide additional material to challenge the superior student. Two short chapters are given to *Analytical Geometry* and *Solid Geometry* in an attempt to interest the pupil in further study of mathematics.

The format is good, type is large and diagrams are clear and attractive.—K. EILEEN BECKETT, Lebanon, Indiana.

Analytic Geometry, L. M. Kells and H. C. Stotz. New York, Prentice-Hall, Inc., 1949. viii +280 pages. $2.85.

The authors state: "The aim of this text is to give such a presentation of the materials of analytic geometry that a student will be led to a deeper understanding of underlying principles while he is learning to apply them in solving the ordinary problems of the subject."

In keeping with this expressed aim, the authors introduce fundamental topics as early as possible and use these ideas repeatedly. Specific illustrations of this early treatment are vectors introduced on page two and translation of axis on page twenty-four during the study of graphs. This book gives an adequate treatment of polar equations and parametric equations as needed for the calculus.

The material is quite readable and the graphs are excellent for both two and three dimensional representation. There are ample problems.

Answers to all problems are included. Appendix I gives reference materials and Appendix II is a table of miscellaneous functions.

The author has written a text which is adaptable to courses of varied lengths.—F. R. Brown, Illinois State Normal University, Normal, Illinois.

Collected Mathematical Papers. George David Birkhoff. New York, American Mathematical Society, 1950. 3 volumes, lvii+754, 983, 897 pp. $18.00.

It is only with the publication of such a collection as this, of Birkhoff's papers, that one begins to get a really accurate idea of the scope of the accomplishment of a great mathematician. The broadness of the mathematical achievements of Birkhoff is already indicated by the several categories into which the collected papers have been divided. Volume one contains papers on boundary value problems and associated Sturmian problems, on differential equations, on difference equations, and four papers on dynamics. Volume two contains papers on dynamics and on physical theories. Volume three contains papers on the four color problems and miscellaneous papers. With relatively few exceptions the papers in all but the section on physical theories and the last section are of a research nature.

The final section contains two papers which appear in Yearbooks of the National Council of Teachers of Mathematics: (1) *A New Approach to Elementary Geometry* (with Ralph Beatley); (2) *Polygonal Forms*. The second paper refers to Birkhoff's theory of aesthetics, there being several other papers on this subject in the final section. The paper on elementary geometry is supplemented by a more detailed exposition in the paper *A set of postulates for plane geometry, based on scale and protractor*. The reviewer thought these papers extremely interesting and sides with Birkhoff in the view that there is no special virtue in sticking to the classical Euclidean approach to geometry in the standard high school course. The student at that level is in no position to really appreciate the postulational approach as typified by Euclid's geometry. It would seem that Birkhoff's method of basing geometry on concepts established in the prior training of the student is much more sound pedagogically. And there is absolutely no virtue in retaining Euclid for merely traditional reasons.

Among the expository papers of the final section the reviewer found *Mathematics: Quantity and Order* extremely interesting. A person with very little technical background can gain a little insight of some of the most profound problems of mathematics by perusal of this paper.

Included in Volume 1 are three obituaries of Birkhoff.—D. H. Potts. Northwestern University, Evanston, Illinois.

An Introduction to Probability Theory and Its Applications. Volume 1. William Feller. New York, John Wiley & Sons, Inc. 1950. xii+ 419 pp. $6.00.

It occasionally happens that when a first-rate mathematician "stoops" to writing a textbook the result is also first-rate. Such is the case in point. The author is one of the leading mathematicians of our day; his product is a highly interesting, skillfully written book which should prove to be greatly used in years to come. His treatment of the subject of probability is entirely mathematical, the speculative philosophical aspects having been properly divorced from the mathematical theory. In this volume the author has confined himself to consideration of "discrete sample spaces," thus requiring of the reader only an elementary mathematical background (a first course in calculus would suffice for much of the material). Problems involving more advanced mathematical techniques have been deferred to a second volume. However, such relatively advanced topics as infinite Markov chains and random walk are included.

From a pedagogical point of view the reviewer thought this book excellent. On the publisher's jacket one finds "a distinguishing feature of the book is the wealth of illustrative examples." This is not an overstatement. Each chapter ends with adequate problem sets. Answers are given in the back of the book.—D. H. Potts, Northwestern University, Evanston, Illinois.

Watch for the program for the 29th Annual Meeting of the NCTM in Pittsburgh, March 28-31, 1951 in the February issue. Make your hotel reservations now!